§7.1 AREA of a REGION between 2 CURVES

Reminder: Definite Integral ⇔ the Area of a Region above the x-axis under a curve

Area of a Region between 2 curves

If \( f \) and \( g \) are continuous on \([a,b]\) and \( g(x) \leq f(x) \) for all \( x \) in \([a,b]\), then the area of the region bounded by the graphs of \( f \) and \( g \) and the vertical lines \( x = a \) and \( x = b \) is:

\[
A = \int_{a}^{b} [f(x) - g(x)] \, dx
\]

Ex. 1: Find the area of the region enclosed by \( y = x \) and \( y = 2 - x^2 \) between \( x = 0 \) and \( x = 2 \).

Do you notice that the 2 above functions intersect twice? Well, finding the Area of a region bounded by 2 intersecting graphs is another common area problem. You first need to find the values of \( a \) and \( b \) to set up the definite integral.
Ex. 2: Find the area of the region bounded by $y = x$ and $y = 2 - x^2$

Solution:
First, find the points of intersection:

If 2 curves **intersect at more than 2 points**, then to find the area of the region between these curves, you do the following:
* find all points of intersection
* see which curve is above the other in each interval determined by the points of intersection
* set up one appropriate definite integral for the area in each interval
* add all the definite integrals

Ex. 3: Find the area of the region bounded by $f(x) = x^3 + x^2 + 1$ and $g(x) = 1 + 3x - x^2$
If the graph of a function of $y$ is a boundary of a region, it is often convenient to use representative rectangles that are **horizontal** and find the bounded area by integrating with respect to $y$:

$$A = \int_{y_1}^{y_2} [(\text{right curve}) - (\text{left curve})] \, dy$$

**Ex. 4:** Find the area of the region bounded by $f(y) = 4 - y^2$ and $g(y) = y - 2$

**Ex. 5:** (Integration as an Accumulation Process)

Find the accumulation function $F$: $F(\alpha) = \int_{-1}^{\alpha} \cos\left(\frac{\pi \theta}{2}\right) \, d\theta$

Now, evaluate:
(a) $F(-1) =$
(b) $F(1/2) =$