7.4 ARC LENGTH and SURFACES of REVOLUTION

**Definition of Arc Length**

Let the function given by $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The **arc length** of $f$ between $a$ and $b$ is

$$s = \int_a^b \sqrt{1 + [f’(x)]^2} \, dx.$$ 

Similarly, for a smooth curve given by $x = g(y)$, the **arc length** of $g$ between $c$ and $d$ is

$$s = \int_c^d \sqrt{1 + [g’(y)]^2} \, dy.$$ 

**Ex 1:** Find the length of the arc of the semicubical parabola $y^2 = x^3$ between the points $(1,1)$ and $(4,8)$. 
**Definition of Surface of Revolution**

If the graph of a continuous function is revolved about a line, the resulting surface is a **surface of revolution**.

**Definition of the Area of a Surface of Revolution**

Let \( y = f(x) \) have a continuous derivative on the interval \([a, b]\). The area \( S \) of the surface of revolution formed by revolving the graph of \( f \) about a horizontal or vertical axis is

\[
S = 2\pi \int_a^b r(x) \sqrt{1 + \left[ f'(x) \right]^2} \, dx
\]

where \( r(x) \) is the distance between the graph of \( f \) and the axis of revolution. If \( x = g(y) \) on the interval \([c, d]\), then the surface area is

\[
S = 2\pi \int_c^d r(y) \sqrt{1 + \left[ g'(y) \right]^2} \, dy
\]

where \( r(y) \) is the distance between the graph of \( g \) and the axis of revolution.