§8.5 PARTIAL FRACTIONS (Partial Decomposition)

Decomposition of $N(x)/D(x)$ into Partial Fractions

1. **Divide if improper**: If $N(x)/D(x)$ is an improper fraction (that is, if the degree of the numerator is greater than or equal to the degree of the denominator), divide the denominator into the numerator to obtain

$$\frac{N(x)}{D(x)} = (\text{a polynomial}) + \frac{N_1(x)}{D(x)}$$

where the degree of $N_1(x)$ is less than the degree of $D(x)$. Then apply Steps 2, 3, and 4 to the proper rational expression $N_1(x)/D(x)$.

2. **Factor denominator**: Completely factor the denominator into factors of the form

$$(px + q)^m \quad \text{and} \quad (ax^2 + bx + c)^n$$

where $ax^2 + bx + c$ is irreducible.

3. **Linear factors**: For each factor of the form $(px + q)^m$, the partial fraction decomposition must include the following sum of $m$ fractions.

$$\frac{A_1}{(px + q)} + \frac{A_2}{(px + q)^2} + \cdots + \frac{A_m}{(px + q)^m}$$

4. **Quadratic factors**: For each factor of the form $(ax^2 + bx + c)^n$, the partial fraction decomposition must include the following sum of $n$ fractions.

$$\frac{B_1}{ax^2 + bx + c} + \frac{B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{B_n}{(ax^2 + bx + c)^n}$$
Guidelines for Solving the Basic Equation

Linear Factors

1. Substitute the roots of the distinct linear factors into the basic equation.

2. For repeated linear factors, use the coefficients determined in guideline 1 to rewrite the basic equation. Then substitute other convenient values of $x$ and solve for the remaining coefficients.

Quadratic Factors

1. Expand the basic equation.

2. Collect terms according to powers of $x$.

3. Equate the coefficients of like powers to obtain a system of linear equations involving $A, B, C$, and so on.

4. Solve the system of linear equations.
§8.6 INTEGRATION by TABLES (& Other INTEGRATION TECHNIQUES)

❖ Integration Tables are in Appendix B (pages A18-A22) – please make sure to correctly identify the expressions “u”, “du”, and the constants “a”, “b”, “n”.

❖ For the curious minds: Use the following substitution to find the integrals in #63, #65, #66 (page 566).

**Substitution for Rational Functions of Sine and Cosine**

For integrals involving rational functions of sine and cosine, the substitution

\[ u = \frac{\sin x}{1 + \cos x} = \tan \frac{x}{2} \]

yields

\[ \cos x = \frac{1 - u^2}{1 + u^2}, \quad \sin x = \frac{2u}{1 + u^2}, \quad \text{and} \quad dx = \frac{2 \, du}{1 + u^2}. \]

The other problems in Exercises 63-70 can be solved by other integration techniques previously learned.