AMATYC SML Fall 2006 – SOLUTIONS

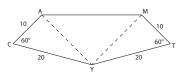
- **1.** Line passes through (0, a) and (a, 0). m = (a 0)/(0 a) = -a/a = -1 (Answer: B)
- **2.** The set is: $\{\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\}$. Of the 10 elements, only the decimal representations of $\frac{1}{3}$ and $\frac{2}{3}$ do not terminate. So 8 of 10 terminate, thus 4/5. (Answer: E)
- **3.** The dimensions of the box are $s \times \frac{36}{s} \times \frac{63}{s}$; If the sides have integer length, the largest value for s is 9 and the smallest value is 1. $V = \frac{2268}{s}$; Minimum value for V is 252, maximum is 2268. (Answer: D)
- 4. Let a be the value of the fifth score. $\bar{x} = \frac{a+388}{5}$. For \bar{x} to be an integer, a must be 2 plus a multiple of 5. 97 works, 92 works, anything less than 92 will not work. (Answer: C)
- **5.** The numbers are 96, 95, 87. (Answer: B)
- 6. P(misses two) \cap [P(makes one) \cup P(misses one and makes one)] = $0.2^2[0.8 + 0.2 \times 0.8] = 0.0384$ (Answer: B)
- **7.** $5a 3b = 0, -10a + 6b = 0, 5c 3d = 0, -10c + 6d = 0 \rightarrow a = \frac{3}{5}b, c = \frac{3}{5}d$. If a, b, c, and d must be positive integers, b = 5 and d = 5 which makes a = 3 and c = 3. (Answer: D)
- 8. Let w be the number of hours worked on weekdays, t for Saturdays, and n for Sundays. w+t+n = 180and 10w + 15t + 20 = 2315. In the first equation, solve for t and substitute into the second equation. $t = 180 - w - n \rightarrow 10w + 15(180 - w - n) = 2315$. Simplify to get w - n = 77. (Answer: B)

9.
$$s(s(1/6)) + S(S(1/3)) = s(\sin\frac{\pi}{6}) + S(\sin^2\frac{\pi}{3}) = s(\frac{1}{2}) + S(\frac{3}{4}) = \sin\frac{\pi}{2} + \sin^2\frac{3\pi}{4} = 1 + \frac{1}{2}$$
 (Answer: D)

- 10. p = 2, q = 17, r = 59, p + q + r = 78 which is even, so one of the primes must be 2. The problem reduces to finding the pairs of primes that sum to 76. All possible other solution sets: $\{2, 3, 73\}, \{2, 5, 71\}, \{2, 23, 53\}, \{2, 29, 47\}$. (Answer: C)
- **11.** (B) $105 = 3 \cdot 5 \cdot 7$. Eliminate all numbers less than 1000 that are divisible by 3, 5 and 7. 333 divisible by 3, 200 66 = 134 divisible by 5 and not 3, 142 47 28 + 9 = 76 divisible by 7 but not 3 or 5. So we get 1000 (333 + 134 + 76) = 457. (Answer: B)
- 12. (B) Let M be the midpoint of \overline{AB} . EM = 12, the altitude of $\triangle ABE$. $\triangle MBE \sim \triangle GCB$ so $\frac{12}{5} = \frac{10}{CG} \rightarrow CG = \frac{25}{6}$. Area = $10^2 (2)(\frac{1}{2})(\frac{25}{6})(10)$.
- **13.** $x^{\log_{25}9} + 9^{\log_{25}x} = 54 \rightarrow (25^{\log_{25}x})^{\log_{25}9} + (25^{\log_{25}9})^{\log_{25}x} = 54 \rightarrow 2 \cdot 9^{\log_{25}x} = 54 \rightarrow 9^{\log_{25}x} = 27 \rightarrow \log_{25}x = \frac{3}{2} \rightarrow x = 125$. Solve $x^3 125x^2 x + 125 = 0$ using factoring by grouping to get $x = \{-1, 1, 125\}$. (Answer: A)
- 14. What is the prob. that in 5 chosen (lost) at random, you have exactly one pair? $P = \frac{8 \times_7 C_3 \times 2^3}{{}_{16}C_5}$. 8 is the number of ways to pick a pair, ${}_7C_3$ is the number of ways to chose three socks that don't match, 2^3 because there are two ways to pick the first non pair, two ways to pick the second and two ways to pick the third. ${}_{16}C_5$ is the number of ways to pick 5 socks from the 16. (Answer: A)
- 15. irrational zeros of h(k(x)) are $x = \frac{3\pm\sqrt{37}}{2}$, irrational zeros of k(h(x)) are $x = \frac{1\pm\sqrt{17}}{4}$ (Answer: C) 16. rational zero of h(k(x)) is $x = \frac{1}{2}$, rational zero of k(h(x)) is $x = -\frac{7}{4}$ (Answer: A)

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17. We can conclude that $\angle C = \angle T = 60^{\circ}$. Draw lines AY and MY and you have triangles with sides in the ratio of 2:1 with a 60° included angle so they must be 30-60-90° triangles. It follows that AYM is a 45-45-90° right triangle. AY = MY = $10\sqrt{3}$. The area is then $2(\frac{1}{2})(10)(10\sqrt{3}) + \frac{1}{2}(10\sqrt{3})^2 \approx 323$ (Answer: D)



- 18. Let the number be ABCD. The rule for divisibility of 11 is A B + C D = [a number divisible by 11]. A B and C D must be even numbers, therefore A B + C D must also be even. Since 22 is not possible, A B + C D = 0. It follows that A B = D C, so the problem reduces to finding the numbers from the set $\{1, 3, 5, 7, 9\}$ that will satisfy this equation. It can be determined that A B can result in only 9 possible outcomes: $0, \pm 2, \pm 4, \pm 6, \text{ and } \pm 8$. There are 5 ways for A B = 0 times 5 ways for D C = 0. There are 4 ways for A B = 2 and four ways for A B = -2. There are 3 ways for A B = 4 and 3 ways for A B = -4. Continue this pattern to get: (5)(5) + (2)(4)(4) + (2)(3)(3) + (2)(2)(2) + (2)(1)(1) = 85. (Answer: B)
- **19.** $3^{2007} = 3^7 \cdot 9^{1000} = 3^7(10-1)^{1000}$ From the binomial expansion formula: $3^7(10-1)^{1000} = 3^7(...+1000 \cdot 10 \cdot (-1)^{999} + (-1)^{1000})$. Looking at the expansion we can see that each term except the last term (which is 1), will be divisible by 100 and therefore multiplying the result of this expansion by 3^7 will not affect the tens digit of 3^7 . $3^7 = 2, 187$ (a tens digit of 8). (Answer: E)

20. $a_2/a_1 = 2$, $a_3/a_2 = 2.5$, $a_4/a_3 = 3$,... $a_n/a_{n-1} = \frac{1}{2}n + 1$ (Answer: E)