1. L: $y = 3x + 3$, M: $y = -\frac{1}{2}x + 3$ (Answer: A)

2. Solve the system: $\begin{cases} 1.05S = L + 1200 \\ 1.01S = L \end{cases}$ (Answer: D)

3. $(ax^2 + bx + c) \div (x + 1) = ax + b - a$ with remainder $c + a - b$. Since the remainder equals zero, use $b = a + c$ and solve for x in $ax + b - a = 0$. (Answer: D)

4. 5 pennies has more than one solution, 10 pennies is too many. Therefore 0 pennies, 9 nickels. (Answer: E)

5. x and y represent the slope of the line. Find the only pair that can't reduce. (Answer: B)

6. Every 4 jumps = 8 numbers. The flea advances 502 numbers in 2008 jumps. $(502 \mod 12) = 8$. (Answer: E)

7. EFGH is a parallelogram. (Answer: D)

8. 15 subsets, 3 give duplicate sums. (Answer: C)

9. $\begin{cases} ax - b = c \\ bx - c = a \end{cases} \Rightarrow \begin{cases} -abx + b^2 = -bc \\ abx - ac = a^2 \end{cases} \Rightarrow b^2 - ac = a^2 - bc \Rightarrow a^2 - b^2 + ac - bc = 0$
$\Rightarrow (a + b)(a - b) + c(a - b) = 0 \Rightarrow a + b + c = 0$ (Answer: A)

10. Vertical asymptote at x = 15, horizontal asymptote at y = 1 (Answer: C)

11. The largest number that would fit the pattern is 989,765. Find the largest multiple of 55 that is less than this number, which is 989,725 and it happens to also fit the pattern. (Answer: C)

12. The power of 6 is suspiciously large – if a is not a power of 2, then it must be 3, since $3^6 = 729 < 2009$, but $5^6 > 2009$. If $a = 3$, that means we must write $2009 - 729 = 1280$ as a sum of powers of 2. The highest power of 2 less than 1280 is 1024, and sure enough, $1280 = 1024+256 = 2^{10}+2^8 = (2^5)^2+(2^4)^2$. Altogether, $2009 = 3^6 + 32^2 + 16^2$, so $a + b + c = 3 + 32 + 16 = 51$. (Answer: E)

13. It will happen when Feb. 1 lands on a Friday in a leap year *and* that day is an even number of weeks away from 2/1/2008. 7 leap years later will be 1461 weeks, so we need to go another 7 leap years or 56 years. The year will be 2064. (Answer: C)

14. A and D have opposite stories so one must be true and the other false. B and E have the same story, and since only two suspects are lying, they must be telling the truth. Therefore C is lying. (Answer: E)

15. Too little information is given, in the sense that not enough answer choices are given – the correct one is missing! Write the first sequence as $a, a+r, a+2r, a+3r, \ldots$ and the second as $b, b+s, b+2s, b+3s, \ldots$. The information about the product sequence amounts to the three equations (A) $ab = 468$, (B) $ab + as + br + rs = 462$, and (C) $ab + 2as + 2br + 4rs = 384$. We are asked for the next term in the product sequence, which is $(a + 3r)(b + 3s) = ab + 3as + 3br + 9rs$. Subtracting (B) from (C), we have $as + br + 3rs = -78$. Multiplying this by 3 and adding $ab = 468$, we find the next term in the sequence is $ab + 3as + 3br + 9rs = 468 + 3(-78) = 234$.

16. $$\frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}} = \frac{\sin\frac{A-B}{2}\cos\frac{A+B}{2}}{\cos\frac{A-B}{2}\sin\frac{A+B}{2}} = \frac{\frac{1}{2}(\sin A - \sin B)}{\frac{1}{2}(\sin A + \sin B)} = \frac{\frac{9}{7}\sin B - \sin B}{\frac{9}{7}\sin B + \sin B} = \frac{1}{8}$$ (Answer: A)

17. The plane separates the original pyramid into the desired solid and a smaller pyramid with height 6 m, similar to the original pyramid. The base of this small pyramid is determined by $\frac{6}{9} = \frac{b_S}{6} \Rightarrow b_S = 4$ m. Therefore $V = V_L - V_S = \frac{1}{3}(6)^2(9) - \frac{1}{3}(4)^2(6) = 76$ (Answer: E)

18. Area of $\triangle$ABC $= (AB)^2 \sin(A)$. Area of $\triangle$DEF $= (DE)^2 \sin(D) = \left(\frac{AB}{2}\right)^2 \sin(2A) = \frac{(AB)^2}{2} \cos(A) \sin(A)$ (Answer: B)

19. The resulting triangle is equilateral with sides of length 50. $A = \frac{1}{2}(50)(25\sqrt{3}) = 625\sqrt{3}$ (Answer: A)

20. The key here is to wonder why they wrote the first term the way they did. If you multiply out the the first two terms, leaving the powers of 2 as powers of 2, you get $(2^2+2^1+1)(2^2-2^1+1) = (2^4+2^2+1)$, which is similar to the next term except for the sign. Suspecting a pattern, go ahead and multiply $(2^4+2^2+1)(2^4-2^2+1)$ and see that you get (2^8+2^4+1). So what we have is a telescoping product (prove it if you like) and the whole thing simplifies to

$$P(k) = (2^{2^{k+2}} + 2^{2^{k+1}} + 1) - 1 = 2^{2^{k+2}} + 2^{2^{k+1}} = 2^{2^{k+1}}(2^{2^{k+1}} + 1) = n(n+1).$$

So we need to find the lowest k for which

$$n + n + 1 = 2n + 1 = 2^{2^{k+1}+1} + 1 \geq 10^{1000}.$$

The left side is odd and the right is even, so this is the same as $2^{2^{k+1}+1} \geq 10^{1000}$. Take $\log = \log_{10}$ of both sides to get $2^{k+1} + 1 \geq 1000/\log 2 = 3321.9$, so we need to have $2^{k+1} \geq 3322$. Take log again to find $k + 1 \geq \log 3322/\log 2 = 11.7$, so $k \geq 10.7$, so k is at least 11. (Answer: C)