1. $(2 \cdot 3) \wedge (2 + 3) = 6^5 = 7776$. (Answer: E)

2. $(1 + 0.20)(1 - 0.10)(1 + 1/6) = \frac{12}{10} \cdot \frac{9}{10} \cdot \frac{7}{6} = \frac{126}{100} = 1 + 0.26$, so 26%. (Answer: B)

3. The sum and difference of the equations are $2ax = 28$ and $-2by = -12$. Set $x = 2$ and $y = 3$ and solve: $a = 7$ and $b = 2 \implies a + b = 9$. (Answer: D)

4. a is at most 3, since $3^6 = 729$ but $4^6 = 4096 > 2011$. If $a = 3$, then $b^2 + c^2 = 2011 - 729 = 1282$. Beginning with $n = 26 \approx \sqrt{1282/2}$, make a table of values $(n, \sqrt{1282 - n^2})$ and quickly find that $29^2 + 21^2 = 1282$, so $a + b + c = 3 + 29 + 21 = 53$. (Answer: D)

5. The number of possible *mixed* shades is the number of distinct values among the fractions R/W with $R = 1, 2, 3, 4$ and $W = 1, 2, 3, 4, 5$. This is the number of such fractions with no commmon prime factor, and these are easy to list: $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{2}{1}, \frac{2}{3}, \frac{2}{5}, \frac{3}{1}, \frac{3}{2}, \frac{3}{4}, \frac{3}{5}, \frac{4}{1}, \frac{4}{3}, \frac{4}{5}$. Including pure red and white, there are 15+2=17 shades possible. (Answer: C)

6. $f(-2) = f(6) = 0$, so set $2 - 2x = -2, 6 \implies x = 2, -2$. (Answer: A)

7. $P_1(t) = Pe^{rt}$ and $P_2(t) = Pe^{-rt} \implies P_1(t)P_2(t) = P^2e^{rt}e^{-rt} = P^2$. (Answer: B)

8. Since $(x + m)(x + n) = x^2 + (m + n)x + mn = x^2 + (m + n)x + 8$, we must have $mn = 8$ and $m + n > 0$, so the only integer possibilities are $\{m, n\} = \{1, 8\}, \{2, 4\}$. Since $b > c$, $b = 1 + 8 = 9$ and $c = 2 + 4 = 6$ and $b - c = 9 - 6 = 3$. (Answer: C)

9. Ed's speed during the n^{th} half hour is $v_n = v_1 - 5(n - 1)$, an arithmetic sequence. In the first three hours he travels $\frac{1}{2}[v_1 + (v_1 - 5) + (v_1 - 10) + (v_1 - 15) + (v_1 - 20) + (v_1 - 25)] = 3v_1 - 37.5$ miles, and in the last 20 minutes he travels $\frac{1}{3}(v_1 - 30) = \frac{1}{3}v_1 - 10$ miles. Since he travels $\frac{10}{3}v_1 - 47.5 = 197.5$ miles in all, $v_1 = 73.5$ and he travels $\frac{1}{2}(73.5 + 68.5 + 63.5 + 58.5) = 132$ miles in the first two hours. (Answer: B)

10. Sun starts with .20(10) liters of antifreeze, removes x of the mixture to obtain $.20(10 - x)$ liters of antifreeze, then adds x liters of antifreeze for $.20(10 - x) + x = 2 + .8x$ liters of antifreeze. This equals $.25(10) = 2.5$, so $2 + .8x = 2.5 \implies x = 5/8 = 0.625$. (Answer: D)

11. $\dfrac{7!}{1!} + \dfrac{7!}{2!} + \cdots + \dfrac{7!}{6!} = 5040 + 2520 + 840 + 210 + 42 + 7 = 8659$. (Answer: E)

12. Add, subtract, and simplify the equations $\implies x + y = 6$ and $y - x = 18 \implies (x, y) = (-6, 12)$ is the point of intersection. The lines have reciprocal slopes so are symmetric with respect to *two* lines through $(-6, 12)$, one with slope 1 and one with slope -1. Since one condition is $m > 0$, the line of symmetry is $y - 12 = 1(x - (-6)) \implies y = 1x + 18$, so $m + b = 1 + 18 = 19$. (Answer: D)

13. Let r be the radius of the circle and x the side length of the square inscribed in the semicircle. Since $r^2 = x^2 + (x/2)^2 = (5/4)x^2$, the smaller square has area $x^2 = (4/5)r^2$. The larger square has side length $2(r/\sqrt{2}) = r\sqrt{2}$, so its area is $45 = (r\sqrt{2})^2 = 2r^2 \implies r^2 = 45/2$, and the smaller square has area $(4/5)r^2 = (4/5)(45/2) = 18$. (Answer: B)

14. Each fold reduces the previous angle by half, so the final right triangle has angle $\pi/8$ adjacent to 157 and opposite the height x, so $x = 157\tan(\pi/8) = 65.03$. (Answer: C)

15. Let $(x, y) = $ (#full boxes, #empty boxes). The rules are that an empty box can either remain empty or be filled with 5 new empty boxes; symbolically, $(x, y) \to (x+1, y-1+5) = (x+1, y+4)$ is allowed. More generally, $(x, y) \to (x+n, y+4n)$ is allowed, so in particular, $(0, 1) \to (18, 1+4\cdot 18) = (18, 73)$ is allowed. (Answer: A)

16. If Al is correct, then Di wins bio, leaving math for Al (since he is correct). Since Di wins bio, she is correct that Bo wins physics, so Bo is wrong and Cy does not win chem; but only chem remains, which is a contradiction. So Al is not correct.
Since Al is not correct, Di does not win bio and Cy is correct about Al not winning math. Since Cy is correct, she does not win chem, so Bo is wrong. Since Al and Bo are wrong, Di must be correct and, since she does not win bio, she must win math. (Answer: D)

17. We can rule out 71, 73, and 74 since these have prime factors > 9. If the three products are P_1, P_2, P_3, then $P_1P_2P_3 = 9!$ and the largest of these is $P \geq \sqrt[3]{9!} = 71.33$, so we rule out 70 and the answer is $72 = \max\{9\cdot 8\cdot 1, 7\cdot 5\cdot 2, 6\cdot 4\cdot 3\}$. (Answer: C)

18. The red chips determine the white chips, so let $x_i =$ the number of red chips in container $i = 1, 2, 3$. We count the number of integer solutions to $x_1 + x_2 + x_3 = 10$, subject to $0 \leq x_1 \leq 6, 0 \leq x_2 \leq 10, 0 \leq x_3 \leq 9$. Ignoring the upper restrictions, there are $C_{10+3-1,3-1} = C_{12,2} = 66$ solutions. There are $C_{3+3-1,3-1} = C_{5,2} = 10$ ways to have $x_1 > 6$ and 1 way to have $x_3 > 9$, so there are 66 - 10 - 1 = 55 valid solutions in all. (Answer: D)

19. $\triangle EBG \sim \triangle CDG \implies \dfrac{x}{72-x} = \dfrac{1}{2} \implies 2x = 72 - x \implies x = 24$. (Answer: D)

20. $\displaystyle\sum_{k=1}^{\infty} kr^{5k-1} = \sum_{k=0}^{\infty}(k+1)r^{5k+4} = r^4\sum_{k=0}^{\infty}(k+1)r^{5k} = r^4\sum_{k=0}^{\infty}(k+1)u^k$, where $u = r^5$. Since $|r| < 1$, the series converges, and there are at least two ways to find the sum:

With calculus: $\displaystyle\sum_{k=0}^{\infty}(k+1)u^k = \frac{d}{du}\sum_{k=0}^{\infty}u^{k+1} = \frac{d}{du}\left(\frac{1}{1-u} - 1\right) = \frac{1}{(1-u)^2}$.

Without calculus:

$$\begin{array}{lccccccccc}
\displaystyle\sum_{k=0}^{\infty}(k+1)u^k & = & 1 & + & 2u & + & 3u^2 & + & \cdots \\
 & = & 1 & + & u & + & u^2 & + & \cdots \\
 & + & & & u & + & u^2 & + & \cdots \\
 & + & & & & & u^2 & + & \cdots \\
 & \cdots & & & & & & &
\end{array}$$

$$= \sum_{k=0}^{\infty}u^k + u\sum_{k=0}^{\infty}u^k + u^2\sum_{k=0}^{\infty}u^k + \cdots = (1 + u + u^2 + \cdots)\sum_{k=0}^{\infty}u^k = \left(\sum_{k=0}^{\infty}u^k\right)^2 = \frac{1}{(1-u)^2}.$$

Since $u = r^5$, the sum of the given series is $\dfrac{r^4}{(1-r^5)^2}$.

Since r is a root of P, $9r^5 + 7r^2 - 9 = 0 \implies 9(r^5 - 1) = -7r^2 \implies 1 - r^5 = \frac{7r^2}{9}$ so the sum is $\dfrac{r^4}{(1-r^5)^2} = \dfrac{r^4}{(7r^2/9)^2} = \dfrac{81}{49}$ and $a + b = 81 + 49 = 130$. (Answer: E)