1. A piano has 88 keys, each either black or white. If two white keys were added, 60% of the keys would be white. The number of black keys is

A. 34 B. 36 C. 38 D. 40 E. 42

2. Two rectangular boxes, one of which is 3 cm by 6 cm by 5 cm, have the same volume. If the other box has integer length edges and a square base with area more than 1 sq cm, find its height in cm.

A. 6 B. 9 C. 10 D. 12 E. 15

3. Map 1 has a scale of 3/4 inch to a mile, and Map 2 has a scale of 7/8 inches to a mile. If the two map distances between Oldville and Newtown add to a total of 52 inches, how far apart in miles are the towns?

A. 28 B. 30 C. 32 D. 36 E. 40

4. Anh, Ana, and Ann have average age A, and Ana, Ann, and Ann's twin Amy have average age B. If Ann is 12 years older than Anh, find B - A.

A. 4 B. 6 C. 8 D. 12 E. 24

5. Americans measure gas mileage in miles per gallon, while Europeans measure it in liters per 100 kilometers. To the nearest tenth, find the European gas mileage equivalent to 25 miles per gallon (1 gal = 3.785 liters; 1 mile = 1.609 kilometers).

A. 9.0 B. 9.1 C. 9.2 D. 9.4 E. 9.6

6. The equation $a^5 + b^2 + c^2 = 2012$ has a solution in positive integers for which b - c is prime. Find a + b + c for this solution.

A. 44 B. 48 C. 52 D. 56 E. 60

7. A number is chosen at random from among all 5-digit numbers containing only the digits 1, 2, 3, 4, and/or 5. Find the probability that the digits are all different.

A. 0.03 B. 0.036 C. 0.0384 D. 0.04 E. 0.0432

8. Bob and Roy leave simultaneously from Alpha and Beta (60 miles apart), headed toward each other at constant speeds, and meet after 1 hour. If Bob left 1/4 hour later, they would meet 6 minutes later. Find the difference in mph of their speeds.

A. 6 B. 12 C. 15 D. 18 E. 21

9. The standard formula to convert degrees Fahrenheit (F) to degrees Celsius (C) is $C = \frac{5}{9}(F - 32)$. The conversion can also be done by adding a constant K to F, multiplying by 5/9, and then subtracting the same constant K. Find K.

A. 32 B. 35 C. 36 D. 40 E. 45

10. In the sequence $\{a_n\}$, a_1 is a 1-digit number. Each a_n is the remainder when $3a_{n-1}$ is divided by 11. If $a_8 = 5$, find a_1.

A. 1 B. 3 C. 4 D. 5 E. 9

11. If f(x) = $\frac{\sqrt{4-x^2}}{x-1}$ and g(x) = $\frac{x+1}{x-1}$, which of the following is in the domain of $\frac{f(x)}{g(x)}$?

A. -3 B. -1 C. 1 D. 3 E. none of these

12. If $\log_2 x$ and $\log_2 y$ are distinct positive integers and $\log_x 2 + \log_y 2 = 0.5$, $xy =$

A. 64 B. 128 C. 256 D. 512 E. 1024

13. How many times do the graphs of y = 3 sin x and y = 3 sin 3x intersect for x in the interval [0, 2012] ?

A. 1920 B. 1921 C. 1922 D. 1923 E. 1924

14. If $P(x) = x^4 - x^3 - 10x^2 + 5x + 25 = Q(x)R(x)$, where Q(x) and R(x) are two nonconstant polynomials with integer coefficients, find Q(4) + R(4).

A. 16 B. 18 C. 20 D. 22 E. 24

15. By NCAA rules a softball diamond is a square ABCD with AB = 60 ft. The pitcher's rubber is 43 ft from home plate (vertex A of the square) on diagonal AC. If point B is first base, the distance from the pitcher's rubber to first base is closest to

A. 42.43 ft B. 42.47 ft C. 43.05 ft D. 43.25 ft E. 43.37 ft

16. For how many integer values of n is $\frac{5n-8}{2n+4}$ an integer?

A. 0 B. 2 C. 4 D. 6 E. an infinite number

17. Let a and b be a pair of positive integers for which $a^2 = 2b^2 - 2$ (4 and 3 is the smallest such pair). Find a - b for the smallest such pair for which a + b > 100.

A. 33 B. 35 C. 37 D. 39 E. 41

18. A sequence of positive integers $\{a_n\}$ is defined by $a_{n+2} = a_{n+1} + a_n$ for $n \geq 1$. If $a_8 = 82$ and $a_{11} = 348$, what is a_9?

A. 129 B. 130 C. 131 D. 132 E. 133

19. Square ABCD with vertices (0, 0), (2, 0), (0, 2), and (2, 2) is rotated 30° around the origin. Find the area common to the original and rotated squares.

A. $\frac{6\sqrt{3}}{5}$ B. $\frac{4\sqrt{3}}{3}$ C. $\frac{3\sqrt{3}}{2}$ D. $\frac{8\sqrt{3}}{5}$ E. $\frac{5\sqrt{3}}{3}$

20. For the equation p + q + r + s = 22, how many solutions in positive integers are there with $p \leq 4$?

A. 650 B. 660 C. 670 D. 680 E. 690

Test #1 **AMATYC Student Mathematics League** October/November 2012

1. B
2. C
3. C
4. A
5. D
6. E
7. C
8. B
9. D
10. B
11. E
12. D
13. C
14. B
15. A
16. D
17. E
18. E
19. B
20. A