1. $\dfrac{w+2}{88+2} = 0.60 \implies \dfrac{w+2}{90} = \dfrac{3}{5} \implies w = 52.$ $b = 88 - 52 \implies b = 36.$ (Answer: B)

2. The volume of the unknown box is the same as the known box which is $3 \times 6 \times 5 = 90$ cm^2. Let b be the length and width of the base and let h be the height, which gives: $b^2h = 90$. Since b must be an integer greater than 1, and h must also be an integer, the only possible solution to this equation is $b = 3$ and $h = 10$. (Answer: C)

3. Let d be the distance between Oldville and Newtown in miles. On Map 1, the distance is represented as $\frac{3}{4}d$ inches and on Map 2 the distance is $\frac{7}{8}d$ inches. $\frac{3}{4}d + \frac{7}{8}d = 52 \implies d = 32$ miles. (Answer: C)

4. Let A_1 be Anh's age, A_2 be Ana's age and A_3 be Ann's age. $(A_1+A_2+A_3)/3 = A \implies A_1+A_2+A_3 = 3A$. Ann and her twin Amy have the same age so $(A_2+2A_3)/3 = B \implies A_2+2A_3 = 3B$. Subtract the first equation from the second to get $A_3 - A_1 = 3(B - A)$. It's given that $A_3 - A_1 = 12$ which leads to $B - A = 4$. (Answer: A)

5. $\dfrac{25 \text{ mi}}{1 \text{ ga}} = \dfrac{25(1.609)\text{ km}}{3.785 \text{ L}} = \dfrac{0.40225 \times 100 \text{ km}}{3.785 \text{ L}} \implies \dfrac{3.785 \text{ L}}{0.40225 \times 100 \text{ km}} \approx 9.4 \text{ L}/100 \text{ km}.$ (Answer: D)

6. $(3, 40, 13)$ and $(3, 37, 20)$ are both solutions but the first one has $b - c = 27$, which is not prime, therefore $a + b + c = 3 + 37 + 20 = 60$. (Answer: E)

Hint: These solutions can be found fairly quickly using the TABLE function on a graphing calculator by first recognizing that a can only be 1, 2, 3, or 4 and then enter `Y = √( 2012 - a ∧ 5 - X ∧ 2)`. Guess values for a then scroll through the table to identify integer values for `Y`.

7. The number of 5-digit numbers containing the digits 1 through 5 is 5^5. The number of ways to rearrange the numbers 1, 2, 3, 4, 5 is 5!. $P = 5!/5^5 = 0.0384$ (Answer: C)

8. Let B be Bob's speed and R be Roy's. $rt = d$ so $B + R = 60$. Also $B(1 + \frac{6}{60} - \frac{1}{4}) + R(1 + \frac{6}{60}) = 60 \implies \frac{17}{20}B + \frac{11}{10}R = 60$. Solve the system to get $B = 24$, $R = 36$. (Answer: B)

9. $\frac{5}{9}(F - 32) = \frac{5}{9}(F + K) - K \implies \frac{5}{9}(-32) = \frac{5}{9}K - K \implies K = 40$ (Answer: D)

10. Trying the possible choices given for a_1 you'll see that the sequence eventually repeats which makes the development of the sequence quicker. For example, when $a_1 = 1$ the sequence is $\{1, 3, 9, 5, 4, 1, 3, 9, ...\}$, as soon as you get back to 1, you know the next numbers in the sequence will repeat. Here $a_8 = 9$ so that doesn't work. However, 5 does occur in this sequence so we suspect if we start at a different number, we might end up with 5 as our 8th element. Sure enough, when $a_1 = 3$, the pattern is the same as above but $a_8 = 5$. (Answer: B)

11. Let $h(x) = \dfrac{f(x)}{g(x)} = \dfrac{(x-1)\sqrt{4-x^2}}{(x-1)(x+1)}$. h has a hole at $x = 1$, a vertical asymptote at $x = -1$ and is not a real number for any x such that $|x| > 2$. (Answer: E)

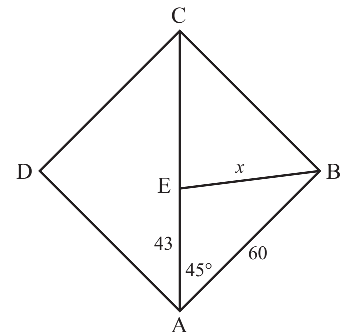
12. The values for xy are $2^3 \cdot 2^6$ (or $2^6 \cdot 2^3$). (Answer: D)

13. The two curves intersect six times for $0 \le x < 2\pi$. 2π goes into 2012, 320 times with some left over. That gives us $320 \times 6 = 1920$ points of intersection, plus the number of times the curves intersect in the remaining range. To determine this remaining range, take $2\pi \times 320$ and subtract from 2012 which is about 1.38. Using a graphing calculator, we can quickly see that the curves intersect two more times for $0 \le x \le 1.38$. (Answer: C)

Note: This can also be done algebraically by determining the six solutions for $0 \le x < 2\pi$ which are $\{0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}\}$ and then proceeding like above noticing that 1.38 is in between $\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

14. Rearranging the terms $P(x)$ can be written: $P(x) = (x^4 - 10x^2 + 25) - (x^3 - 5x)$ which can be factored by grouping to get: $P(x) = (x^2 - 5)^2 - x(x^2 - 5) = (x^2 - 5)(x^2 - x - 5) = Q(x)R(x)$. $Q(4) + R(4) = 11 + 7 = 18$. (Answer: B)

15. Label the point at the pitcher's rubber E creating $\triangle$ABE and let x be side EB (the length we are trying to find). We know $\angle$EAB = 45° so from the law of cosines: $x^2 = 43^2 + 60^2 - 2\cdot 43\cdot 60\cos 45° \implies x = \sqrt{43^2 + 60^2 - 43\cdot 60\cdot\sqrt{2}} \approx 42.43$. (Answer: A)



16. Using long division you can rewrite $\dfrac{5n-8}{2n+4}$ as $\dfrac{1}{2}\left(5 - \dfrac{18}{n+2}\right)$. This expression will be an integer when $5 - \dfrac{18}{n+2}$ is an even integer, which will occur only when $\dfrac{18}{n+2}$ is an odd integer. $\dfrac{18}{n+2}$ is an odd integer when $n+2$ is a factor of 18 that is paired with an odd factor of 18 or when $n+2$ in the set $\{\pm 2, \pm 6, \pm 18\}$. Therefore all possible values for n are $-20, -8, -4, 0, 4$, and 16. (Answer: D)

Hint: These solutions may be found quicker by using the TABLE function on a graphing calculator. Enter `Y = (5X - 8)/(2X + 4)` and scan through the table to find integer values for `Y`.

17. Use the TABLE feature on a calculator with $Y = \sqrt{2X^2 - 2}$ to identify integer pairs $(a, b) = (Y, X)$ and quickly determine that the first with $a + b > 100$ is $(a, b) = (140, 99)$, so $a - b = 41$. (Answer: E)

18. $348 = a_{11} = a_{10} + a_9 = (a_9 + a_8) + a_9 = 2a_9 + 82 \implies a_9 = (348 - 82)/2 = 133$. (Answer: E)

19. Rotating counterclockwise, the common region is a "kite" with obvious vertices at $O = (0, 0)$ and $A = (0, 2)$, slightly less obvious vertex at $C = (2\cos 30°, 2\sin 30°) = (\sqrt{3}, 1)$, and fourth vertex $B = (x, 2)$ where the top of the unrotated square intersects a side of the rotated square. The midpoint of A and C is $M = (\sqrt{3}/2, 3/2)$ and, since O, M, and B are collinear, $x/2 = \sqrt{3}/3 \implies B = (2\sqrt{3}/3, 2)$. The area of a kite is half the product of its diagonals, which in this case is $\frac{1}{2}\overline{OB}\cdot\overline{AC} = \frac{1}{2}\sqrt{(2\sqrt{3}/3 - 0)^2 + (2-0)^2}\cdot\sqrt{(\sqrt{3}-0)^2 + (2-1)^2} = 4/\sqrt{3} = 4\sqrt{3}/3$. (Answer: B)

20. Equivalently, determine the number of non-negative integer solutions to $a+b+c+d = 18$ with $a \le 3$. Without the restriction on a, the number of solutions is $C(18+3, 3) = \frac{21!}{3!18!} = 1330$, because each solution corresponds to a choice of 3 counters to be "dividers" in a row of 21 counters: the number of counters to the left of the first divider is then a, the number between the first two dividers is b, and so on. The number of solutions with $a \ge 4$ is the number of unrestricted solutions to $a' + b + c + d = 14$, which is $C(14+3, 3) = \frac{17!}{3!14!} = 680$. So in all, the number of solutions is $1330 - 680 = 650$. (Answer: A)