1. $1E_0 = 1.25D_0$ and $\dfrac{E_1}{D_1} = 1.1\left(\dfrac{E_0}{D_0}\right) = 1.1(1.25)$, so $2.2D_1 = \dfrac{2.2}{1.1(1.25)}E_1 = 1.6E_1$. (Answer: C)

2. $Equation1 + 2 \cdot Equation2 \implies y = a + 2b$. (Answer: D)

3. $\log(\log(\log 10^{10^{10}})) = \log(\log(10^{10})) = \log(10) = 1$. (Answer: B)

4. $7777 = 2525 + 5252, 77777 = 38839 + 38938, 777777 = 123456 + 654321$, and $7777777 = 3888839 + 3888938$, so only 777 could not equal the sum. (Answer: B)

5. The lines are perpendicular, so $\dfrac{-A}{B} = \dfrac{A}{C}$. Since $A \neq 0$, $C = -B$ and the sum of the equations is $(A - B)x + (A + B)y = D + E$, so $A - B = 6$ and $A + B = 10$. Solve this system to find $A = 8$ and $B = 2$, so the lines have slopes -4 and $\dfrac{1}{4}$. (Answer: D)

6. TYC + SML = 4M4, so the possibilities for $\{C, L\}$ are $\{1, 3\}, \{5, 9\}$, and $\{6, 8\}$. If $\{C, L\} = \{1, 3\}$, then $Y = 0$ and there is no valid option for $\{T, S\}$. If $\{C, L\} = \{5, 9\}$, then $Y = 9$, which is not valid. If $\{C, L\} = \{6, 8\}$, then $Y = 9$ and $\{T, S\} = \{1, 2\}$, so M can be 0, 3, 5, or 7. (Answer: B)

7. If $m > 0$ is the minimum, then the numbers are m, m, m, n, and $2m$ for some $m \leq n \leq 2m$. The mean of these is $m + (n/5)$ and must equal n, so $n = 5m/4$ and the ratio of the maximum to the mean is $2m/(5m/4) = 8/5 = 1.6$. (Answer: A)

8. L passes through the midpoint $(4, 7)$ and is perpendicular to the segment with slope $\dfrac{10 - 4}{2 - 6} = -3/2$, so its slope is $2/3$ and the equation is $\dfrac{y - 7}{x - 4} = \dfrac{2}{3} \implies 3y - 2x = 13$. (Answer: D)

9. By the power property and change of base formula, the left side is equal to $(2\log_8 x)\left(\dfrac{1}{\log_8 x}\right)^2 = \dfrac{2}{\log_8 x}$, which equals 1 iff $x = 8^2 = 64$. (Answer: D)

10. If Al is a knight, then it is true that Bo is a knight, and therefore it is true that Cy is a knave; this is consistent with Cy's false statment about Al and Bo, so this case is possible. If Al is a knave, then Bo is not a knight, so Bo is a knave, and therefore Cy is no a knave, Cy is a knight; this is NOT consistent with Cy's false statement about Al and Bo, so this case is impossible. Thus, Al and Bo must be knights and Cy must be a knave. (Answer: C)

11. Use the TABLE feature on a graphing calculator and values $a = \lfloor 2013^{1/4} \rfloor = 6, 5, 4, \ldots$, to search for integer values of $c = Y = \sqrt{2013 - a^4 - 2X^2} = \sqrt{2013 - a^4 - 2b^2}$, where $X = b$ is an integer. It does not take long to find the solution $(a, b, c) = (5, 26, 6)$, so $a + b + c = 37$. (Answer: B)

12. Compute the first few terms to see that, starting with a_3, they are all equal to $\frac{1}{2}(a_1 + a_2)$, so the sum of the first 24 terms equals $12(a_1 + a_2)$. It is intuitively obvious that all the terms starting with a_3 are equal, since each is the average of all the previous terms, and by always adding the previous average to the list, the average will never change, so the terms will not change. To prove this, observe that $a_{n+1} = \dfrac{a_1 + \cdots + a_{n-1} + a_n}{n} = \dfrac{a_1 + \cdots + a_{n-1}}{n-1} \cdot \dfrac{n-1}{n} + \dfrac{a_n}{n} = a_n\left(\dfrac{n-1}{n}\right) + a_n\left(\dfrac{1}{n}\right) = a_n$ for all $n \geq 3$, so $a_3 = a_4 = a_5 \ldots$. (Answer: E)

13. $x^2 = 4y^2 + 81 \iff (x-2y)(x+2y) = 81$, so an integer solution satisfies the system $x - 2y = N$ and $x + 2y = 81/N$, where N is a (positive or negative) factor of 81. For such an N, the corresponding solution is $x = \frac{1}{2}(N + 81/N)$ and $y = \frac{1}{4}(81/N - N)$. Since all ten possibilities for $N (\pm 1, \pm 3, \pm 9, \pm 27, \pm 81)$ lead in this way to integer solutions, there are 10 elements in S. (Answer: E)

14. Consider the graph of $y = ||x| - 6|$, which looks like a W with vertices at $(-6, 0), (0, 6)$, and $(6, 0)$. It is necessary to find k so that the horizonal lines $y = k + 2$ and $y = k - 2$ intersect the W in a total of exactly 5 points. The only horizontal line that intersects the W in three points is $y = 6$, so either $k - 2 = 6$ or $k + 2 = 6$. Only the first case works, leading to $k = 8$ and the line $y = k + 2 = 10$ which intersects the W in two points. The solutions are $x = -16, -12, 0, 12, 16$. (Answer: 8)

15. There are $b - 1$ unreduced proper fractions with denominator $b > 1$, so the number of such fractions with $b \leq n$ for some $n > 1$ is $1 + 2 + \cdots + (n - 1) = n(n - 1)/2$. Estimate $n \approx \sqrt{2 \cdot 2013} \approx 63.5$ and find there are exactly 2016 such fractions with denominator ≤ 64. The last few of these are $\ldots, \frac{60}{64}, \frac{61}{64}, \frac{62}{64}, \frac{63}{64}$, so the 2013th is $\frac{60}{64}$, so $a + b = 124$. (Answer: A)

16. Draw a picture and let $x = AF$. $\triangle FGD \sim \triangle FAB$, so $\dfrac{x}{12} = \dfrac{AF}{FG} = \dfrac{AB}{DG} = \dfrac{DG + GC}{DG} = 1 + \dfrac{CG}{DG}$. Also, $\triangle ADG \sim \triangle ECG$, so $\dfrac{15}{12 + x} = \dfrac{EG}{AG} = \dfrac{CG}{DG}$. Eliminate $\dfrac{DG}{CG}$ from these two equations to obtain $\dfrac{x}{12} = 1 + \dfrac{15}{12 + x}$, and solve to find $AF = x = 18$. (Answer: B)

17. In this problem, the game does NOT start over every 3 coin flips: a single sequence of coin flips is generated until someone's pattern of 3 is observed. (For example, if Mo chooses THT and the first 5 coin flips are HTTHT, then Mo wins on the 5th flip.) Therefore, if Mo chooses THH, then Ha wins in only one way, if HHH appears immediately in the first 3 flips. If Mo chooses anything besides THH, then Ha has other ways to win. Since Mo's chances are best when Ha's chances are worst, and having only one way to win is the worst that can happen to Ha, Mo should choose THH. (Answer: A)

18. If Mo chooses THH, the only way Ha wins is immediately, and there is a $1/8$ chance of this happening. Since someone must win, that leaves Mo with a $7/8$ chance of winning. (Answer: E)

19. Let $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ and suppose p/q is a rational solution in lowest terms, so $a(p/q)^4 + b(p/q)^3 + c(p/q)^2 + d(p/q) + e = 0 \implies ap^4 + bp^3q + cp^2q^2 + dpq^3 + eq^4 = 0$. There are an odd number of terms, so p and q can not both be odd, or else all the terms are odd, which means the sum is odd, but the sum is zero which is even. If p is even, then the first four terms are even and the last is odd, since p and q have no common factors. This means even + odd makes zero, which is also impossible. Similarly, if q is even and p is odd, so it just plain can't be done, son. (Answer: A)

20. Let $x = CF$ and $y = BE$. Since $\triangle CDF$ and $\triangle ADE$ have equal areas, $4x = 2(4-y) \implies y = 2(2-x)$. Since $\triangle CDF$ and $\triangle BEF$ have equal areas, $4x = (2 - x)y = 2(2 - x)^2 \implies x = 3 \pm \sqrt{5}$ and, since $x < 4, x = 3 - \sqrt{5}$. Thus, the desired ratio is $\dfrac{8 - 3 \cdot 2(3 - \sqrt{5})}{2(3 - \sqrt{5})} = \sqrt{5}$. (Answer: B)