

1. A store sells 10 apples for \$6 or 20 apples for \$10. How much would you save by buying 25 apples at the 20 apple rate instead of the 10 apple rate?  
A. \$2    B. \$2.50    C. \$3.00    D. \$3.25    E. \$4
2. The two lines with equations  $ax + 12y = 6$  and  $ax - 3y = 12$  ( $a \geq 0$ ) are perpendicular. Find  $a$ .  
A. 2    B. 3    C. 4    D. 6    E. 8
3. Working together, Bob, Celeste, and Daria take one hour to paint a fence. Working alone, Bob would take  $b$  hours, Celeste would take  $c$  hours, and Daria would take  $d$  hours, where  $b$ ,  $c$ , and  $d$  are positive integers and  $b < c < d$ . Find  $b + c + d$ .  
A. 9    B. 10    C. 11    D. 12    E. 13
4. The students in Ms. Nguyen's 8<sup>th</sup> grade math class can be seated in rows of 4 or 5, each time with exactly the same number of seats in each row, but when seated in rows of 6, one row has exactly 2 fewer students than all the other rows. If 4 new students join the class, in how many equal rows could her students now be seated?  
A. 7    B. 8    C. 9    D. 10    E. 11
5. Let  $p$  and  $q$  be two constants for which the equation  $2x + p = q$  has the solution  $x = 12$ . Find the solution to the equation  $3x + q = p$ .  
A. -18    B. -8    C. -4    D. 8    E. 18
6. My piggy bank has 42 coins worth exactly \$1.00. If it has at least one quarter, dime, nickel, and penny, find the total number of dimes and nickels.  
A. 3    B. 4    C. 5    D. 6    E. 7
7. For a third-degree polynomial  $P(x)$ ,  $P(-1) = P(1) = P(2) = 2$  and  $P(3) = 10$ . Find  $P(4)$ .  
A. 20    B. 24    C. 28    D. 30    E. 32
8. A rectangular box has a long diagonal of length 17, a face diagonal of length 15, and an edge of length 9, all sharing a common endpoint. Find the volume of the box.  
A. 648    B. 672    C. 720    D. 768    E. 864
9. Ed spends \$25 on 3 DVD's, each costing a whole number of dollars. The first DVD costs more than twice the second, but less than three times the third; the second DVD costs more than the third. Find the cost of the third DVD.  
A. \$2    B. \$3    C. \$4    D. \$5    E. \$6
10. Let  $f(x) = x^2$ . For positive numbers  $a$  and  $b$ , find the  $y$ -intercept of the line through  $(a, f(a))$  and  $(-b, f(-b))$ .  
A.  $ab$     B.  $a + b$     C.  $a - b$     D.  $b - a$     E.  $a^2 - b^2$
11. Let  $\triangle ABC$  be a right triangle with integer length sides whose perimeter is numerically equal to its area. What is the largest possible value for its perimeter?  
A. 12    B. 24    C. 30    D. 36    E. 40

12. For  $n$  a nonnegative integer, find all positive integer values of  $a$  and  $b$  such that  $4n + 9 = a^2$  and  $9n + 1 = b^2$ , and then find the sum of all such  $a$ 's and  $b$ 's.  
A. 4    B. 10    C. 32    D. 36    E. 42
13. Knights always tell the truth, and knaves always lie. A knight and  $12N - 1$  other people (each one either a knight or a knave) sit in a circle, and each one says, "Exactly one of my two immediate neighbors is a knave." How many knaves are in the circle?  
A.  $2N$     B.  $3N$     C.  $4N$     D.  $6N$     E. It depends on  $N$
14. Let  $S$  be the set consisting of the number 258131, together with all the other different numbers formed by rearranging its digits. Find the sum of the elements of  $S$ .  
A. 122222100    B. 133333200    C. 166666500    D. 199999800    E. 266666400
15. In right triangle  $ABC$  (right angle at  $C$ ), points  $D$  and  $E$  lie on the hypotenuse so that  $AD = DE = BE$ . If  $CD = 10 \sin x$  and  $CE = 10 \cos x$  for some value  $x$ , find  $AB$ .  
A.  $6\sqrt{5}$     B. 15    C.  $5\sqrt{10}$     D. 20    E.  $15\sqrt{2}$
16. A deck of cards contains 4 cards with value \$13, 4 cards with value \$5, 4 cards with value \$1, and the other 40 cards with value \$0. If the first 12 cards randomly dealt from the deck have a value of \$38, find the probability to the nearest hundred-millionth that the next 6 cards randomly dealt from the remaining cards have total value  $\geq$  \$38.  
A. 0.00000026    B. 0.00000028    C. 0.00000030    D. 0.00000032    E. 0.00000036
17. The equation  $a^5 + b^3 + c^2 = 2015$  has two solutions in positive integers. In one of these,  $ab$  is a factor of  $c$ . Find  $\frac{c}{ab}$ .    A. 3    B. 4    C. 5    D. 6    E. 7
18. Anh and Carlos randomly and independently choose positive integers  $A$  and  $C$ , respectively, both  $\leq 50$ . If  $A < C$ , Anh collects  $C - A$  dollars; otherwise there is no payoff to either player. What is Anh's average payoff if this process is repeated many times?  
A. \$7.75    B. \$7.83    C. \$8.00    D. \$8.33    E. \$8.45
19. Three dice, a blue, a red, and a yellow, are rolled, giving a sum of 13. The red and yellow dice are rerolled, with a new sum on all 3 dice of 6. The blue and red dice are rerolled, giving a sum on all 3 dice of 15. Finally, the blue and yellow dice are rerolled, giving a sum on all 3 dice of 7. Find the sum of the 3 numbers rolled on the yellow die.  
A. 8    B. 9    C. 10    D. 11    E. 12
20. If  $\log_4 m = \log_6 n = \log_9(m+n)$ , then  $\frac{m}{n}$  can be represented in the form  $\frac{a + \sqrt{b}}{c}$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $c$  is a prime number. Find  $a + b + c$ .  
A. 6    B. 7    C. 8    D. 9    E. 10

Test #1    **AMATYC Student Mathematics League**    October/November 2015

1. B
2. D
3. C
4. E
5. B
6. D
7. E
8. E
9. D
10. A
11. C
12. D
13. C
14. B
15. A
16. A
17. E
18. D
19. D
20. A