

1. Alice takes her favorite number, adds 5 to it, multiplies the answer by 10, subtracts 20 from the result, and then drops the final 0. If Alice's (correct) answer is 9, what is her favorite number?
- A. 5 B. 6 C. 7 D. 8 E. 9
2. The two distinct lines with equations $ax + y = c$ and $x + y = c$ intersect at the point $(b - 2, 4)$. Find $b + c$. A. 0 B. 2 C. 4 D. 6 E. 8
3. George runs from home to school at 9 mph, then jogs from school to the store at 4.5 mph, and finally bikes back home. If home, school, and store are equidistant, and George's average speed is 7.5 mph, find his biking speed in mph.
- A. 10.5 B. 12 C. 12.5 D. 15 E. 18
4. A triangle with integer-length sides has no two sides equal. What is its least possible perimeter? A. 6 B. 7 C. 8 D. 9 E. 10
5. If both the length and width of a 30 by 50 rectangle are increased by the same amount d , the new rectangle's area is exactly 8 times the original rectangle's area. In which interval below does d lie?
- A. $[40, 54]$ B. $[55, 70]$ C. $[71, 85]$ D. $[86, 99]$ E. $[100, 110]$
6. A trapezoid has consecutive sides of length 8, 10, and 12 in that order. Find the largest possible area of such a trapezoid rounded down to the nearest integer, and write your answer in the corresponding blank on the answer sheet.
7. Each member of a group of 6 people is either a knight, who always tells the truth, or a knave, who always lies. The first person says, "There is at least one knave in this group," and the n th person ($2 \leq n \leq 6$) says that there are at least n knaves in the group. How many knaves are there? A. 1 B. 2 C. 3 D. 4 E. 5
8. A roll of paper towels forms a hollow cylinder with outer diameter 5", inner diameter $1 \frac{3}{4}$ ", and height 11". If the roll contains 120 sheets 11" by 10.4" of uniform thickness with no space between adjacent sheets, find the thickness of a sheet to the nearest ten-thousandth of an inch.
- A. 0.0138" B. 0.0142" C. 0.0146" D. 0.0150" E. 0.015"
9. How many ordered pairs (k, n) of integers are solutions to $k^2 - 2016 = 3^n$?
- A. 0 B. 1 C. 2 D. 3 E. 4
10. The radius of a quarter-circle is k times the radius of a semicircle. If the perimeter of the semicircle (including its diameter) equals the perimeter of the quarter-circle (including its two radii), find k to the nearest hundredth.
- A. 1.44 B. 1.50 C. 1.56 D. 1.60 E. 1.64
11. For how many rational values of x is $P(x) = |x^2 - 28x + 160|$ a prime number?
- A. 0 B. 1 C. 2 D. 3 E. 4

12. Suppose that the two 3-digit numbers abc and cba differ by 792 ($c, a \neq 0$). If the middle digit $b = 6$, find the sum of the two numbers.

- A. 928 B. 1128 C. 1130 D. 1190 E. 1228

13. A party is held for only people born in April. If the probability of being born on any particular day in April is the same as any other day, find the least number of people to attend which makes the probability $> 50\%$ that two people at the party were born on the same day of the month. A. 7 B. 8 C. 9 D. 12. E. 15

14. How many solutions does the system $\begin{cases} x^2 + 15xy = x + 15y \\ y^2 - xy = 15x + y \end{cases}$ have?

- A. 1 B. 2 C. 3 D. 4 E. 5

15. An infinite geometric series has common ratio $2/3$ and sum S . If the first, third, and all other odd terms are doubled, and the second, fourth, and all other even terms are halved, what is the sum of the new series?

- A. $1.6S$ B. $1.4S$ C. $1.2S$ D. $0.8S$ E. $0.6S$

16. Assuming that ties are allowed, what is the number of ways five contestants can be ranked, if the order of tied contestants doesn't matter? For example, A might be 1st, B and D tied for 2nd, E 4th, and C 5th, or A and D might be tied for 1st and B, C, and E tied for 3rd. Write your answer in the corresponding blank on the answer sheet.

17. If (a, b, c) is a solution to the system of equations $\begin{cases} 5x + 3y + z = 2 \\ 6x + 5y + 4z = 8 \end{cases}$, find $a+b+c$.

- A. -2 B. -1 C. 0 D. 1 E. 2

18. The number whose base 15 representation is the 3-digit number abc has the 4-digit representation $1abc$ in base 6. Find $a + b$.

- A. 4 B. 5 C. 6 D. 7 E. 8

19. A and B play the following game: A draws a chip from a bag containing 4 green and 8 red chips. If the chip is green, A wins; if not, A returns the chip to the bag, and B draws a chip. If the chip is red, B wins; otherwise, B returns the chip to the bag, and the alternating draws repeat. Find the probability that A wins the game.

- A. $\frac{1}{7}$ B. $\frac{3}{14}$ C. $\frac{2}{7}$ D. $\frac{5}{14}$ E. $\frac{3}{7}$

20. How many integer solutions does the system $\begin{cases} a^2 + b^2 + c^2 + d^2 = 2500 \\ (a + 50)(b + 50) = cd \end{cases}$ have?

- A. 12 B. 14 C. 16 D. 18 E. 20

Test #1 **AMATYC Student Mathematics League** October/November 2016

1. B
2. D
3. D
4. D
5. B
6. 120
7. C
8. A
9. C
10. A
11. E
12. C
13. A
14. D
15. B
16. 541
17. E
18. A
19. E
20. E