1. For real numbers a and b, define an operation Δ as $a\Delta b = ab^2 - |a|$. Find $[(-2)\Delta 5]\Delta(-1)$.
A. -104 B. -96 C. -53 D. 0 E. 96

2. Let $n = (1/a) + (1/b) + (1/c)$ where a, b, and c are all positive integers. What is the largest possible value of n that is less than 1?
A. 9/10 B. 11/12 C. 19/20 D. 41/42 E. 63/64

3. Michael is playing a game that involves two quarters (25 cents each), three dimes (10 cents each), one nickel (5 cents), and four pennies (1 cent each). He flips each of the coins once, and wins all of the coins that land heads up. If all of the coins are fair coins (the probability of landing heads up is ½ and the probability of landing tails up is also ½), what is the probability (to the nearest hundredth) that he will win at least fifty cents?
A. 0.40 B. 0.41 C. 0.42 D. 0.46 E. 0.50

4. Find the sum of all of the distinct positive five digit numbers that can be formed by permuting the digits 1, 3, 5, 7, and 8 (13578, 58371, 83517, 18753, etc.).
A. 1,599,984 B. 6,066,540 C. 6,399,936 D. 15,999,840 E. 31,999,680

5. There are 150 socks in a bin: 30 blue, 10 pink, 20 green, 40 black, and 50 white. Jerry randomly pulls socks out of the drawer, one at a time, and does not replace them. Let m be the minimum number of socks that he would need to pull out to guarantee that he has at least one matching (same color) pair. Let M be the minimum number of socks that he must pull out to guarantee that he has at least one sock of each color. Find the product of m and M.
A. 30 B. 66 C. 350 D. 705 E. 846

6. Assume that $\sin(x) + \cos(x) = 1/4$. What is the value of $\sin^3(x) + \cos^3(x)$?
A. 5/26 B. 11/32 C. 31/64 D. 47/128 E. 59/256

7. Let x, y, z be positive integers such that $x^2 + y^2 + z^7 = 2017$. Find $x + y + z$.
A. 59 B. 60 C. 61 D. 62 F. 63

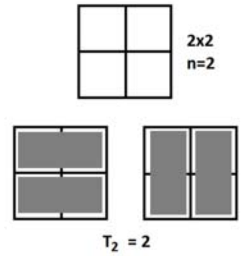
8. How many different ordered 4-tuples of nonnegative integers (a, b, c, d) satisfy the inequality $a + b + c + d \leq 14$?
A. 816 B. 2380 C. 3060 D. 3468 E. 3876

9. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says "I am not a spy.", Y says "X is a knave.", and Z says "Y is a spy." Which of the following correctly identifies all three people?

A.	B.	C.	D.	E.
X is the spy.	X is the spy.	X is the knight.	X is the knight	X is the knave.
Y is the knight.	Y is the knave.	Y is the knave.	Y is the spy.	Y is the spy.
Z is the knave.	Z is the knight.	Z is the spy.	Z is the knave.	Z is the knight.

10. Suppose a, b, and c are integers. What is the sum of the reciprocals of the five complex solutions of the equation $x^5 + ax^4 + bx^3 + cx^2 - 12x + 8 = 0$?
A. $-\sqrt{3}/2$ B. -1 C. $2/\sqrt{3}$ D. 4/7 E. 3/2

11. Let T_n be the number of different ways that a $2\times n$ grid can be covered by n indistinguishable dominos (1×2 rectangles) with no overlaps or gaps. For example, $T_2 = 2$ is illustrated on the right. Find $T_1 + T_2 + T_3 + T_4 + T_5 + T_6$.



A. 19 B. 32 C. 36 D. 53 E. 64

12. In ΔABC, AB = AC = 25 and BC = 14. The perpendicular distances from a point P in the interior of ΔABC to each of the three sides are equal. Find this distance.
A. 9/2 B. 19/4 C. 5 D. 21/4 E. 11/2

13. A coin has probability p of heads and $1 - p$ of tails. If flipped 3 times, it has probability 1/2 of producing three flips with the same result (either 3 heads or 3 tails). Find $p(1 - p)$.
A. 1/12 B. 1/8 C. 1/6 D. 1/3 E. 1/2

14. Consider a data set that consists of positive integers less than 51. There is exactly one 1 in the data set, and every other integer appears twice as many times as its predecessor appears (so there are exactly two 2s, exactly four 3s, exactly eight 4s, exactly sixteen 5s, etc.) What is the median of this data set?
A. 48 B. 49 C. 49.5 D. 50 E. 51

15. Suppose $g\left(\frac{1}{x}\right) = \frac{x^2}{2+x}$. Find $g(g(3))$.
A. 441/23 B. 49/9 C. 9/5 D. 81/95 E. 25/207

16. Let N be the greatest 3-digit positive integer that divides all 4-digit numbers with identical digits (those of the form aaaa). Let M be the greatest positive integer that <u>cannot</u> be written as $3x + 7y$ for some nonnegative integers x and y. Find M+N.
A. 99 B. 110 C. 112 D. 120 E. 121

17. Consider $ax^2 + bx + c = 0$, where a is a nonzero rational number and b and c are real numbers. Determine which (if any) of the following statements are true for all possible values of a, b, and c.
I. If both solutions to this equation are rational, then $b^2 - 4ac$ must be equal to the square of a rational number.
II. If $b^2 - 4ac$ is equal to the square of a rational number, then both solutions to this equation must be rational.
A. Both B. Only I C. Only II D. Neither E. Cannot be determined

18. Find the area of a semicircle inscribed in an equilateral triangle of side length 5. The diameter of the semicircle lies on one side of the triangle with its center at the midpoint of that side, and the semicircle is tangent to the other two sides.
A. $29\pi/8$ B. 2π C. $75\pi/32$ D. $25\pi/4$ E. $75\pi/8$

19. Suppose that $f(x) = \frac{x^2-16}{ax+b}$ for some real numbers a and b, and that $f(x)$ has an oblique asymptote of $y = 3x + 7$. Find $f(-3)$.
A. 65/9 B. 63/16 C. 35/9 D. -7/16 E. -35/9

20. How many positive integers less than or equal to 1000 have an equal number of even and odd factors? For example, 10 would be counted since it has two odd (1 and 5) and two even (2 and 10) factors.
A. 100 B. 125 C. 200 D. 250 E. 500

Test #1 **AMATYC Student Mathematics League** October/November 2017

1. A
2. D
3. B
4. C
5. E
6. D
7. A
8. C
9. C
10. E
11. B
12. D
13. C
14. D
15. A
16. C
17. B
18. C
19. B
20. D