1. It takes Kim twice as long to run 2,255 decimeters as it takes Cara to run 1/10 of a mile. They start 1 kilometer apart and begin running toward each other. How far, to the nearest meter, will Kim have run when they meet? Assume that 1 mile = 1.61 kilometers.
A. 357 m B. 412 m C. 467 m D. 489 m E. 588 m

2. Replace each letter in **ONE + ONE = TWO** with a base-10 digit so that identical letters are replaced by identical digits and different letters are replaced with different digits, **T** is the only odd digit, and **O** cannot be zero. What is the value of **N**?
A. 0 B. 2 C. 4 D. 6 E. 8

3. Which of the following numbers has the greatest value?
A. 2^{1000} B. 6^{500} C. 30^{200} D. 50^{100} E. 1000^{75}

4. The equation $a^2 + b^2 + c^5 = 2019$ has exactly one solution where a, b, and c are positive integers with $a > b$. Find $a + b + c$ for this solution.
A. 56 B. 57 C. 58 D. 59 E. 60

5. Let M be the smallest positive integer that has a remainder of 2 when divided by 3 and has a remainder of 4 when divided by 5. Let N be the smallest positive integer that has a remainder of 6 when divided by 7 and has a remainder of 8 when divided by 9. Find M+N.
A. 70 B. 72 C. 74 D. 76 E. 78

6. There are 200 closed lockers numbered 1-200 in a locker room. Student 1 goes in and opens each locker, then student 2 goes in and closes every other locker (2, 4, 6, 8…). Student 3 then changes the state (opens it if it is closed, closes it if it is open) of every third locker (3, 6, 9, 12…), then student 4 does the same for every fourth locker (4, 8, 12, 16…). This continues until 200 students have gone through, with the n^{th} student changing the state of all lockers numbered with a multiple of n. How many lockers are open at the end?
A. 14 B. 24 C. 48 D. 64 E. 100

7. In some contexts, a function is defined to be *linear* if for all elements x and y in the domain and for all real numbers a, $f(ax) = af(x)$ and $f(x + y) = f(x) + f(y)$. Using <u>this</u> definition of linear, how many of the following functions from $\mathbb{R}$ to $\mathbb{R}$ are linear?

$f_1(x) = 3x$ $f_2(x) = 3x + 2$ $f_3(x) = 2$ $f_4(x) = 0$

A. 0 B. 1 C. 2 D. 3 E. 4

8. Find the sum of all base-10 eight-digit (the first digit cannot be zero) numbers that contain no digits other than 0 or 1 (for example: 10100101, 10000000, 11111111).
A. 711,111,104 B. 1,010,101,010 C. 1,031,111,104 D. 1,351,111,104 E. 1,422,222,208

9. Find $|a - b|$ if a and b are the two real solutions to $(f \circ f \circ f)(x) = 1$ for $f(x) = 2x^2 + 28x + 91$.
A. $\sqrt[8]{2}/4$ B. $\sqrt[8]{2}/2$ C. $\sqrt{2}/2$ D. $\sqrt[8]{2}$ E. $\sqrt{2}$

10. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says, "I am a knight." Y says, "X is telling the truth." Z says, "I am a spy." Which of the following correctly identifies all three people?

A.	B.	C.	D.	E.
X is the spy.	X is the spy.	X is the knight.	X is the knight	X is the knave.
Y is the knight.	Y is the knave.	Y is the knave.	Y is the spy.	Y is the spy.
Z is the knave.	Z is the knight.	Z is the spy.	Z is the knave.	Z is the knight.

11. Let M be the sum of the solutions to $e^{-x}\sin x - e^{-x}\cos x = 0$, where $0 \le x < 2\pi$. Find csc M.

A. -1 B. $-\sqrt{3}/2$ C. 1 D. $2/\sqrt{3}$ E. 2

12. Consider the following function in polar coordinates: $r = \frac{2}{1+0.5cos\theta}$. Which of the following best describes the graph of this fuction?

A. Line B. Two Lines C. Hyperbola D. Parabola E. Ellipse

13. A biased die is rolled until two 1s are rolled in succession, or until a 1 and then a 2 are rolled in succession (in that order). The die lands on 1 with probability 50%, on 2 with probability 20%, and on something else with probability 30%. What is the probability that the rolling will end with successive 1s?

A. 1/3 B. 1/2 C. 4/7 D. 2/3 E. 5/7

14. Find the sum of all complex (both real and nonreal) zeros of $f(x) = \frac{x^3 - \frac{1}{8}}{x - \frac{1}{2}}$.

A. $-1/2 - i\sqrt{3}/2$ B. -1/2 C. 0 D. 1/2 E. $1/2 + i\sqrt{3}/2$

15. How many ordered lists (a, b, c, d, e, f) of nonnegative integers satisfy $a + b + c + d + e + f = 12$?

A. 5304 B. 5544 C. 6160 D. 6188 E. 6468

16. In parallelogram $ABCD$, $\overline{BC}$ is extended beyond point C to point E. Points F and G are the points of intersection of $\overline{AE}$ with $\overline{BD}$ and $\overline{CD}$, respectively. If FG = 12 and EG =15, then find AF.

A. 16 B. 18 C. 20 D. 24 E. 27

17. If the graphs of the functions $f(x) = b(x-m)^2 + n$ and $g(x) = x - m$ intersect, then what is the greatest possible value of bn?

A. 1/4 B. 1/2 C. 3/4 D. 1 E. 2

18. At a school, 69% of Math Club members are also in the Physics Club, and 79% of Math Club members are also on the Quiz Team. Consider the percentage, P, of Math Club members who are both in the Physics Club and also on the Quiz Team. Based on the given data alone, we can find a percentage M and a percentage N that will guarantee that M ≤ P ≤ N. What is the sum of the largest possible value for M and the smallest possible value for N?

A. 52% B. 90% C. 117% D. 127% E. 148%

19. Some children are playing a game that uses a regular octagon *ABCDEFGH*. There are pennies on some of the sides: 1 on $\overline{AB}$, $\overline{BC}$, and $\overline{EF}$; 3 on $\overline{CD}$; 2 on $\overline{DE}$; and none on $\overline{FG}$, $\overline{GH}$, and $\overline{HA}$. Each child, in turn, may add a penny to each of two adjacent sides (for example, a child may add a penny to $\overline{AB}$ and a penny to $\overline{BC}$), but no other changes are permitted. Their goal is to reach a state where all sides have the same number of pennies. If S is the smallest number of turns needed, which inequality does S satisy?

A. This is impossible B. $S \le 8$ C. $8 < S \le 15$ D. $15 < S \le 25$ E. $25 < S$

20. Five distinct integers *a, b, c, d, e* are to be ordered from least to greatest. You are told that *e, d, c, b, a* has at least 3 of the 5 values correctly placed; *e, b, c, d, a* has an odd number of the values correctly placed; and *a, d, c, b, e* is not the solution. You can choose 3 letters and learn their order from least to greatest. Which 3 should you choose to guarantee that the ordering of all 5 numbers can be correctly determined?

A. *a, b, d* B. *a, b, e* C. *b, c, d* D. *b, c, e* E. *c, d, e*

1. B
2. E
3. B
4. B
5. D
6. A
7. C
8. D
9. E
10. D
11. A
12. E
13. E
14. B
15. D
16. B
17. A
18. C
19. A
20. C