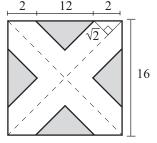
Test #2 AMATYC Student Mathematics League Answers

- 1. (E) g(f(f(1))) = g(f(2)) = g(5) = 25
- 2. (D) From the previous average, if T represents the total miles, 39000 = T/n. With the new car, $36400 = T/(n+1) \Rightarrow T = 39000n = 36400(n+1) \Rightarrow n = 14$
- 3. (C) $\log x, \log x^2, \log x^3, \log x^4, \dots = \log x, 2 \log x, 3 \log x, 4 \log x, \dots$
- 4. (C) Let the numbers, in order, be $a_1, a_2, ..., a_7$. We know $a_4 = 20$. The largest possible value for a_7 is achieved by setting the other unknown integers to their smallest possible values: $a_1 = 1, a_2 = 2, a_3 = 3, a_5 = 21, a_6 = 22$. The average is 20 so the sum must be $140 \Rightarrow 69 + a_7 = 170 \Rightarrow a_7 = 71$
- 5. (C) Since the fraction is a terminating decimal, the prime factors of AT must contain only 2's and 5's. Of the choices, only 16 and 25 remain. Since AM < AT and they have no common factors, trial and error is reasonable. AM = 21, AT = 25, TC = 84
- 6. (D) P = 20 is achieved by tearing out one row of four stamps connected to one additional stamp from an adjacent row. p = 14 is achieved by tearing out one column of five stamps.

7. (A)
$$\log_{st} e^{5.4} = \frac{\ln e^{5.4}}{\ln st} = \frac{5.4}{\ln s + \ln t} = \frac{5.4}{0.6 + 0.9} = 3.6$$

- 8. (B) The symmetry requires f(-2) = -3. 6 is 8 units from -2 so f(6) = -3. The symmetry implies f(4) = -f(-4), but the period requires f(4) = f(-4). It follows that f(4) = 0.
- 9. (D) The graphs don't intersect when the system $\begin{cases} x+y=k\\ xy=k \end{cases}$ has no solution. By substitution, $x+\frac{k}{x}=k \Rightarrow x^2-kx+k=0$. This equation has no solution when its discriminant is negative, $k^2-4k < 0$, which has three integer solutions, 1, 2, and 3.
- 10. (A) The area representing all points $\sqrt{2}$ units from the diagonals is the four isosceles right triangles shaded in the figure shown here. The hypotenuse of each is 12 so each leg is $6\sqrt{2}$. The probability is the shaded area divided by the total area of the square.
- 11. (B) The one-to-one function f will only cross its inverse when y = x. a = 4, b = 12, c = -8.

12. (A)
$$\cos(\arctan(x)) = x \Rightarrow \frac{1}{\sqrt{1+x^2}} = x \Rightarrow x^4 + x^2 - 1 = 0 \Rightarrow x^2 = \frac{-1+\sqrt{5}}{2}$$



13. D

14. B

15. (D) The number of acres can be represented: $A\left(\frac{3}{4}\right)^n + 1500\left(\frac{3}{4}\right)^{n-1} + 1500\left(\frac{3}{4}\right)^{n-2} + 1500\left(\frac{3}{4}\right)^{n-3} + \dots + 1500 = A\left(\frac{3}{4}\right)^n + 1500\sum_{i=0}^{n-1}\left(\frac{3}{4}\right)^i$. As $n \to \infty$, this sum approaches 6000.

16.
$$(B)$$

17. (E)
$$f(x) = \frac{(x-4)(x+1)}{x+1} = x-4, x \neq -1 \Rightarrow f^{-1}(x) = x+4, x \neq -5 \Rightarrow f^{-1}(x) = \frac{(x+4)(x+5)}{x+5}$$

18. (C)

- 19. (D) The pentagon must be regular. It consists of 5 congruent triangles with height 3 (the radius of the circle). $A = 5(\frac{1}{2}bh) \Rightarrow 42 = 5(\frac{1}{2}b(3)) \Rightarrow b = 5.6 \Rightarrow$ the perimeter is 28 cm.
- 20. (E) Take the tangent of both sides, use the sum of angles formula for tangent and solve for x. x = 3.