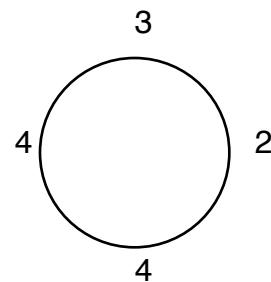


1. Let $P(x) = x^3 - 2x^2 + 3x - 4$. Find the largest prime factor of $P(4) - P(2)$.
- A. 17 B. 19 C. 23 D. 29 E. 31
2. A circle of radius 2 and center E is inscribed inside square ABCD. Find the area that is inside $\triangle ABE$ but outside the circle.
- A. $\pi - 3$ B. $\pi/2 - 1$ C. $4 - \pi$ D. $\pi - 2$ E. $3 - \pi/2$
3. The unique solution to the equation $ax + b = 10$ is $x = 2$, and the unique solution to the equation $bx + a = 8$ is $x = 3$. Find $a + b$.
- A. $\frac{26}{5}$ B. $\frac{28}{5}$ C. 6 D. $\frac{32}{5}$ E. $\frac{34}{5}$
4. The solution to the inequality $\frac{x+1}{x-3} \geq 2$ is
- A. $[-1, 3)$ B. $[-1, 3]$ C. $(3, 7)$ D. $[3, 7)$ E. $(3, 7]$
5. Let a_0, a_1, \dots be an arithmetic sequence with $a_0 = 2$, $a_3 = a_1^2 - 8$, and $a_5 > 0$. Find a_3 .
- A. 4 B. 6 C. 8 D. 10 E. 12
6. All solutions to the equation $a^3 + b^3 + c^2 = 2010$ (a, b, c positive integers) have the same value for $a + b$. Find this value of $a + b$.
- A. 11 B. 12 C. 13 D. 14 E. 15
7. If $z = a + bi$ (a, b real) and $z^2 = 21 - 20i$, $|a| + |b| =$ A. 7 B. 8 C. 9 D. 10 E. 11
8. A point C is chosen on the line segment AB such that $\frac{AC}{BC} = \frac{BC}{5 \cdot AB}$. Find $\frac{AC}{BC}$.
- A. $\frac{-5 + 3\sqrt{5}}{10}$ B. $\frac{-1 + \sqrt{21}}{10}$ C. $\frac{-1 + \sqrt{5}}{10}$ D. $\frac{-5 + \sqrt{29}}{10}$ E. $\frac{5 - \sqrt{5}}{10}$
9. Let $[x]$ represent the greatest integer $\leq x$. Find $\sum_{n=1}^{2010} [\log_5 n]$.
- A. 7256 B. 7260 C. 7262 D. 7263 E. 7264
10. If you roll three fair dice, what is the probability that the product of the three numbers rolled is prime?
- A. $\frac{1}{36}$ B. $\frac{1}{24}$ C. $\frac{1}{18}$ D. $\frac{1}{8}$ E. $\frac{1}{4}$
11. Let $S = (0, 0)$, $M = (10, 0)$, and $L = (10, 10)$, and rotate $\triangle SML$ 30° counterclockwise around S. Find the area of the union of the triangles to the nearest square unit.
- A. 79 B. 81 C. 83 D. 85 E. 87
12. Three faces of a rectangular box that share a common vertex have areas 48, 50, and 54. Find the volume of the box.
- A. 360 B. 364 C. 372 D. 376 E. 384

13. A *multiplicative magic square* (MMS) is a square array of positive integers in which the product of each row, column, and long diagonal is the same. The 16 positive factors of 2010 can be formed into a 4x4 MMS. What is the common product of every row, column, and diagonal? Write your answer in the corresponding blank on the answer sheet.
14. For a function $f(x)$, let $f^2(x) = f(f(x))$, $f^3(x) = f(f(f(x)))$, and so on. For the function $f(x) = \sqrt{\frac{x^2 + 1}{x^2 - 1}}$ on the domain $(-\infty, -1) \cup (1, +\infty)$, $f^{2010}(x) =$
- A. x B. $|x|$ C. x^2 D. $\frac{1}{x}$ E. $\frac{1}{x^2}$
15. You have 4 red, 4 white, and 4 blue identical dinner plates. In how many different ways can you set a square table with one plate on each side if two settings are different only if you cannot rotate the table to make the settings match?
- A. 21 B. 24 C. 27 D. 30 E. 36
16. A 100 m long railroad rail lies flat along level ground, fastened at both ends. Heat causes the rail to expand by 1% and rise into a circular arc. To the nearest meter, how far above the ground is the midpoint of the rail?
- A. 0 B. 2 C. 4 D. 6 E. 7
17. Two sides of a triangle have lengths 25 and 20, and the median to the third side has length 19.5. Find the length of the third side.
- A. 22.5 B. 23 C. 23.5 D. 24 E. 24.5
18. A palindrome is a positive integer like 11, 313, and 5445 which reads the same left to right and right to left. If a number is chosen at random from all four-digit palindromes, find the probability that it is divisible by 7.
- A. $\frac{1}{7}$ B. $\frac{1}{6}$ C. $\frac{2}{11}$ D. $\frac{1}{5}$ E. $\frac{2}{7}$
19. For positive integer values of m and n , the largest value of n for which the system $\begin{cases} a + b = m \\ a^2 + b^2 = n \\ a^3 + b^3 = m + n \end{cases}$ has solutions is A. 3 B. 8 C. 12 D. 24 E. 36
20. A game is played using the 4 piles of chips shown. A move consists of choosing two adjacent piles around the circle and removing at least one chip from at least one of the two piles. The winner is the player who removes the last chip. If 2W 3N means remove 2 chips from the west pile and 3 chips from the north pile, which move guarantees a win for the current player?



- A. 2W 1S B. 2W 1N C. 1W 2S D. 2E 0S E. 1W 4S

Test #2

AMATYC Student Mathematics League

February/March 2010

1. B
2. C
3. B
4. E
5. C
6. D
7. A
8. A
9. E
10. B
11. E
12. A
13. 4,040,100
14. B
15. B
16. D
17. B
18. D
19. D
20. A