1. $P(4) - P(2) = 40 - 2 = 38 = 2 \cdot 19$, so 19. (Answer: B)

2. This is one-fourth the total area inside the square and outside the circle: $\frac{1}{4}[4^2 - \pi(2)^2] = 4 - \pi$. (Answer: C)

3. Solve the system of equations $a(2) + b = 10$ and $b(3) + a = 8$ to find $(a, b) = \frac{1}{5}(22, 6)$, so $a + b = \frac{28}{5}$. (Answer: B)

4. 3 is not a solution and 7 is a solution, which leaves $(3, 7]$ as the only option. Or do it the hard way: Find boundary points of the solution set by finding where the expression is undefined ($x = 3$) and where equality holds ($x = 7$), then plug in test points between and beyond these points to determine the solution set. (Answer: E)

5. With d the common difference, $a_1 = 2+d$ and $a_3 = 2+3d$, so $a_3 = a_1^2 - 8 \implies 2+3d = (2+d)^2 - 8 \implies (d+3)(d-2) = 0 \implies d = -3$ or 2. Since $a_5 = 2+5d > 0$, $d = 2 \implies a_3 = 2+3(2) = 8$. (Answer: C)

6. $12^3 < 2010 < 13^3$, so a brute force search for integer values of $c = \sqrt{2010 - a^3 - b^3}$ with $1 \le a, b \le 12$ is feasible with a calculator, especially using the TABLE feature. (For example: Alternately store the values $1, 2, \ldots$ in the variable A, and look for integers in the table of values for $Y = \sqrt{2010 - A^3 - X^3}$, where X ranges over $1, 2, \ldots, 12$. Even better, if you have the ability to program a matrix, create the 12×12 matrix with ij^{th} entry equal to $\sqrt{2010 - i^3 - j^3}$ and look for integers.) The only positive integer solutions are $(a, b, c) = (5, 9, 34)$ and $(9, 5, 34)$, so $a + b = 14$. (Answer: D)

7. $z^2 = (a + bi)^2 = a^2 - b^2 + 2abi$, so $a^2 - b^2 = (a - b)(a + b) = 21$ and $2ab = -20 \implies ab = -10$. By inspection, $(a, b) = (5, -2)$ is a solution, so $|a| + |b| = 5 + 2 = 7$. (Answer: A)

8. Only the ratio matters, so assume that $\overline{BC} = 1$ and let $x = \overline{AC} = \frac{\overline{AC}}{\overline{BC}} = \frac{BC}{5 \cdot AB} = \frac{1}{5(x+1)} \implies 5x^2 + 5x - 1 = 0$ which, since $x > 0$, means $x = \frac{-5+\sqrt{45}}{10} = \frac{3\sqrt{5}-5}{10}$. (Answer: A)

9. $\lfloor \log_5 n \rfloor = k \iff 5^k \le n < 5^{k+1}$, so the sum is $4(0) + (24 - 4)(1) + (124 - 24)(2) + (624 - 124)(3) + (2010 - 624)(4) = 20 + 200 + 1500 + 5544 = 7264$. (Answer: E)

10. One die must be prime and the others 1; there are 3 choices for the prime (2, 3, or 5), and 3 choices for which die is prime, so 9 ways this can happen. There are 6^3 outcomes possible, so the probability is $9/6^3 = 1/24$. (Answer: B)

11. M and L are on the line $y = x$. If M and L rotate to P and Q, respectively, then $P = 10(\cos 30°, \sin 30°) = 5(\sqrt{3}, 1)$ and the line through P and Q has slope $-\cot 30° = -\sqrt{3}$, so P and Q are on the line $y - 5 = -\sqrt{3}(x - 5\sqrt{3})$, which intersects the line $y = x$ at $R = 10(\sqrt{3} - 1, \sqrt{3} - 1)$. $\overline{PR} = \sqrt{(10\sqrt{3} - 10 - 5\sqrt{3})^2 + (10\sqrt{3} - 10 - 5)^2} = 10\sqrt{7 - 4\sqrt{3}}$, and $\triangle SPQ$ has a right angle at P, so $area(\triangle SPQ) = \frac{1}{2}(10)(10\sqrt{7 - 4\sqrt{3}}) = 50\sqrt{7 - 4\sqrt{3}}$. The area of the union of the triangles is the sum of their areas minus the area of their intersection, which is $2(\frac{1}{2})(10 \cdot 10) - 50\sqrt{7 - 4\sqrt{3}} = 100 - 50\sqrt{7 - 4\sqrt{3}} \approx 86.6 \approx 87$. (Answer: E)

12. $V = \sqrt{V^2} = \sqrt{L^2W^2H^2} = \sqrt{(LW)(WH)(HL)} = \sqrt{(48)(50)(54)} = 360$. (Answer: A)

13. Since 2010 is not a perfect square, its factors pair off: $P_1Q_1 = \cdots = P_8Q_8 = 2010$. If R is the common product of any row or column, then the product of all the rows is $R^4 = P_1Q_1P_2Q_2 \cdots P_8Q_8 = 2010^8 \implies R = 2010^2 = 4040100$. (Answer: 4,040,100)

14. Let $g(x) = f^2(x) = \sqrt{x^2} = |x|$, for $|x| > 1$; then $f^{2010}(x) = g^{1005}(x) = |x|$. (Answer: B)

15. There are 3 arrangements with only one color, $3(1+2+1) = 12$ arrangements with only two colors, and $3(2+1) = 9$ arrangements of all three colors, so 24 arrangements in all. (Answer: B)

16. Let the arc subtend angle 2θ (in radians) from a circle of radius R, so $2R\theta = 101 \implies R\theta = 50.5$. The center C of the circle, the midpoint M of the unexpanded rail, and one end E of the rail form a right triangle $\triangle CEM$. Deduce from this triangle that $50 = \overline{ME} = R\sin\theta$, and that the height to be found is $h = R - \overline{MC} = R(1-\cos\theta)$. To solve for θ, divide the equation $R\theta = 50.5$ by the equation $R\sin\theta = 50$ to obtain $\frac{\theta}{\sin\theta} = 1.01$, and use a graphing calculator to find the solution $\theta \approx 0.244$ radians. From this, calculate $R = 50.5/\theta \approx 206.885$ and $h = R(1-\cos\theta) \approx 6.13$. (Answer: D)

17. Attach a congruent copy of the triangle along the unknown side to form a parallelogram with sides of length 20 and 25, and diagonals of length $2(19.5) = 39$ and the unknown side length, x. For any parallelogram, the formula $d_1^2 + d_2^2 = 2(s_1^2 + s_2^2)$ is true; apply the formula in this case to obtain $x^2 + 39^2 = 2(20^2 + 25^2) \implies x^2 = 529 \implies x = 23$. (Answer: B)

18. Every 4-digit palindrome is of the form $abba$ and there are 90 of these – 9 choices for a and 10 choices for b. Since $abba = a(1001) + b(110) = a(7\cdot 143) + b(7\cdot 15 + 5)$, $abba$ is divisible by 7 iff $5b$ is, i.e., $b = 0, 7$. Thus, there are only 18 4-digit palindromes divisible by 7 – 9 choices for a and 2 choices for b – and the probability is $\frac{18}{90} = \frac{1}{5}$. (Answer: D)

19. $m+n = a^3+b^3 = (a+b)(a^2+b^2-ab) = (a+b)[(a^2+b^2) - \frac{1}{2}((a+b)^2 - (a^2+b^2))] = m[n - \frac{1}{2}(m^2-n)] \implies 2m + 2n = 2mn - m^3 + mn \implies (3m-2)n = m(m^2+2) \implies n = \frac{m(m^2+2)}{3m-2}$. For exam purposes, since this expression for n increases with m, just plug in positive integer values of m until this value exceeds the highest option for n, which is 36; $m = 8$ results in $n = 24$, but $m = 9$ results in a non-integer, and $m = 10$ results in a non-integer > 36, so the largest integer value for n is 24. As for an actual proof this is highest, it follows by polynomial division that $n = \frac{1}{27}\left(9m^2 + 6m + 22 + \frac{44}{3m-2}\right)$; in particular, for n to be an integer, it is necessary that $3m-2$ be a factor of 44, and the highest integer m for which this is true is $m = 8 \implies n = 24$. (Note that the system with $m = 8$ and $n = 24$ has two *complex* solutions, $a = 4 \pm 2i$ and $b = \overline{a}$. The highest integer n for which the system has *real* solutions is $n = 3$, with either $m = 1$ or $m = 2$.) (Answer: D)

20. Under the assumption that this is a 2-player game (not explicitly stated in the problem), 2W 1S is the only move by the current player that will guarantee victory if he/she continues to play correctly. This is because it is the only move that makes the table symmetric with respect to the rules of the game, in the sense that after this move, N and S will both have 3, and W and E will both have 2. This means that no matter what the opponent does next, the current player can mirror it on his/her next move; for example, if the opponent next plays 1W 2N, the current player can then play the equal but opposite move, 1E 2S. By continuing to play this way, the current player forces the opponent to always leave the table with either W $\neq$ E, N $\neq$ S, or both; in particular, the opponent can never leave the table completely empty, so can never win. Since at least one piece must be removed on every turn, the last piece must get taken at some point, and since the opponent can never do this if the current player plays correctly, the current player must eventually take the last piece and win. (Answer: A)