1. Only the percentage is asked for, so assume Anh starts with 100 units of pie and ends up with 60; meanwhile, Ed starts with x units and ends up with $0.8x = 2(60) \implies x = 150$. (Answer: D)

2. $a\#b = a(b^2+1)$, so $250 = (a\#b)\#3 = (a(b^2+1))\#3 = a(b^2+1)(3^2+1) = 10a(b^2+1) \implies a(b^2+1) = 25$. Since $a, b > 0$ are integers, the only solution is $(a, b) = (5, 2) \implies a + b = 7$. (Answer: B)

3. Let A_n = #ways Alicia can climb n stairs. $A_4 = 6$ is given and $A_1 = 1, A_2 = 2$, and $A_3 = 3$ are easy to find. If $n > 4$, Alicia either begins with a step of size 1 and has A_{n-1} ways to continue, she begins with a step of size 2 and has A_{n-2} ways to continue, or she begins with a step of size 4 and has A_{n-4} ways to continue. Thus, $A_n = A_{n-1} + A_{n-2} + A_{n-4}$ for $n > 4$. Start with $n = 10$ and repeatedly apply this formula to find A_{10} in terms of A_1, A_2, A_3, and A_4: $A_{10} = A_9 + A_8 + A_6 = (A_8+A_7+A_5)+A_8+A_6 = 2A_8+A_7+A_6+A_5 = 2(A_7+A_6+A_4)+A_7+A_6+A_5 = 3A_7+3A_6+A_5+2A_4 = \cdots = 18A_4 + 13A_3 + 6A_2 + 10A_1 = 18(6) + 13(3) + 6(2) + 10(1) = 169$. (Answer: E)

4. The sum starting at n is $6n+15$, which should be a perfect cube, so use a calculator to create a table of values for $n = Y = (X^3 - 15)/6$ and look for integers to quickly find $6(119) + 15 = 729 = 9^3$ and $6(560) + 15 = 3375 = 15^3$, so the desired sum is $119 + 560 = 679$. (Answer: A)

5. If S has first term a and common ratio r, then the sum of S is $\dfrac{a}{1-r} = 6$, the sum of the square series is $\dfrac{a^2}{1-r^2} = 15$, and the sum of the opposite series is $\dfrac{a}{1+r} = \dfrac{\frac{a}{1+r}\cdot\frac{a}{1-r}}{\frac{a}{1-r}} = \dfrac{\frac{a^2}{1-r^2}}{\frac{a}{1-r}} = \dfrac{15}{6} = 2.5$. (Answer: B)

6. If the original numbers are $x_1 \le x_2 \le \cdots \le x_{10}$, then the original median is $\frac{x_5+x_6}{2} = 6$ and, since adding 15 to the set raises the median, the new median is $x_6 = 8 \implies x_5 = 4$. Since 7 is between 4 and 8, 7 is the final median. (Answer: E)

7. It is easy to see that a covering is possible with 1×3 tiles, but impossible with 1×4 tiles or 2×3 tiles. It is also impossible with 1×2 tiles, since if the squares on the board are colored like a chess board, there will be 25 of one color and 23 of the other, but a covering by 1×2 tiles requires an equal number of each color, since each tile covers exactly one square of each color. (Answer: B)

8. For $ML \ge 13$, $\angle L$ and $\angle S$ are acute, so the triangle is obtuse iff $\angle M$ is. By the law of cosines, $a^2 + b^2 = c^2 + 2ab\cos C$, with $a = 12, b = 17, c = SL$, and $C = \angle M$, $\cos(\angle M) = \dfrac{12^2 + 17^2 - SL^2}{2(12)(17)}$. Thus, $\angle M$ is obtuse iff $\cos(\angle M) < 0$ iff $SL > \sqrt{12^2 + 17^2} \approx 20.8$, so the answer is 21. (Answer: C)

9. Four conditions are given, so try $P(x) = ax^3 + bx^2 + cx + d$ for some a, b, c, d. Use the given values to obtain four equations: $P(0) = d = 3, P(1) = a + b + c + d = 8, P(2) = 8a + 4b + 2c + d = 39$, and $P(3) = 27a + 9b + 3c + d = 144$. Use $d = 3$ to reduce to a 3×3 system and solve to find $(a, b, c, d) = (8, -11, 8, 3)$. This is not valid, since $-11 < 0$, but for any M, $P(x) = 8x^3 - 11x^2 + 8x + 3 + Mx(x-1)(x-2)(x-3)$ also has the given values at $x = 0, 1, 2, 3$. Expand to $P(x) = Mx^4 + (8 - 6M)x^3 + (11M - 11)x^2 + (8 - 6M)x + 3$, which has non-negative integer coefficients iff $M = 1$, so $P(x) = x^4 + 2x^3 + 2x + 3$ works and $P(-2) = -1$. (Answer: E)

10. 113 is the lowest such prime, so N should start with 113. 131 is the lowest such prime that starts with 13, so N should begin with 1131. 311 is the lowest prime that starts with 31, so N should begin with 11311. And so on, to find the smallest N is 1131131131, which has last two digits 31. (Answer: E)

11. Let $a, a+d, a+2d, \ldots$ and $b, br, br^2, \ldots$ be the unknown arithmetic and geometric sequences, so $ab = 96, (a+d)br = 180, (a+2d)br^2 = 324, (a+3d)br^3 = 567$, and the unknown next product is $x = (a+4d)br^4$. Multiply each of these equations by r and subtract from the next one to obtain the "bdr-equations", $bdr = 180 - 96r, bdr^2 = 324 - 180r, bdr^3 = 567 - 324r$, and $bdr^4 = x - 567r$. Since $r = \frac{bdr^2}{bdr} = \frac{324-180r}{180-96r}$ and $r = \frac{bdr^3}{bdr^2} = \frac{567-324r}{324-180r}$, equate these and simplify to obtain $4(9-5r)^2 = 3(7-4r)(15-8r) \implies 4r^2 - 12r + 9 = 0 \implies r = \frac{3}{2}$. The last two bdr-equations imply $x = 567r + bdr^4 = 567r + r(bdr^3) = 567r + r(567 - 324r) = 1134r - 324r^2 = 1134(3/2) - 324(3/2)^2 = 972$. (Answer: B)

12. Let $c = \log_x y$, so $\log_y x = c^{-1}$ and $c + c^{-1} = 2.9 \implies c^2 - 2.9c + 1 = 0 \implies c = \frac{2}{5}$ or $\frac{5}{2}$. By symmetry, the choice is irrelevant, so take $c = \frac{2}{5} \implies \log_x y = \frac{2}{5} \implies y = x^{2/5} \implies 128 = xy = x^{7/5} \implies x = 128^{5/7} = 32 \implies y = 128/32 = 4 \implies x + y = 36$. (Answer: B)

13. a must be 1, 2, 3, or 4, so $b^2 + c^2$ must be 2010, 1979, 1768, or 987. The sum of two squares has remainder 0, 1, or 2 when divided by 4, so only 2010 and 1768 are possible. Use the table feature on a calculator with $Y = \sqrt{1768 - X^2}$ to quickly find that $2^2 + 42^2 = 1768$, so $(a, b, c) = (3, 2, 42)$ is a solution with two primes, and the nonprime is 42. (Answer: C)

14. Use a table to make a list of values $Y = 17X$ and hunt for palindromes, OR note that the 4-digit palindrome $abba = a(1001) + b(110) = a(17 \cdot 59 - 2) + b(17 \cdot 6 + 8) = 17(59a + 6b) + 2(4b - a)$ is divisible by 17 iff $4b - a$ is. The only possibilities for this are $(a, b) = (2, 9), (3, 5), (4, 1), (7, 6)$, and $(8, 2)$, corresponding to the five palindromes 2992, 3553, 4114, 7667, and 8228. (Answer: C)

15. There are 7 choices for which number to omit and five ways to arrange the rest: Using 1-6 are chosen, these are $\substack{456\\123}$, $\substack{346\\125}$, $\substack{356\\124}$, $\substack{256\\134}$, and $\substack{246\\135}$. So there are $35 = 5 \cdot 7$ arrangements in all. (Answer: D)

16. $160 = a_7 = a_6 + a_5 = 2a_5 + a_4 = 3a_4 + 2a_3 = 5a_3 + 3a_2 = 8a_2 + 5a_1$. By divisibility considerations, a_1 is a multiple of 8 and a_2 is a multiple of 5 and, since the sequence is increasing, the only possibility is $a_1 = 8$ and $a_2 = 15$, so the sequence is 8, 15, 23, 38, 61, 99, 160, and $a_8 = 259$. (Answer: C)

17. Use polynomial division to find that $n^2 + 7 = (n+4)(n-4) + 23$, so the fraction is NOT in lowest terms precisely when 23 is a factor of $n + 4$, i.e. $n = 23k - 4$ for some $k = 1, 2, 3 \ldots$. $n = 23k - 4 \leq 2011 \implies k \leq 87.6$, so there are 87 such integers. (Answer: C)

18. The probability that k pennies are chosen is the coefficient of x^k in the expansion of

$$\left(\frac{1}{3}x + \frac{2}{3}\right)\left(\frac{1}{5}x + \frac{4}{5}\right)\cdots\left(\frac{1}{21}x + \frac{20}{21}\right) = \frac{(x+2)(x+4)\cdots(x+20)}{3 \cdot 5 \cdots 21}.$$

Direct calculation is difficult, but for the test it is easy to eliminate options B, C, and E, since neither 2 nor 23 are in the denominator. Since the numerators of the coefficients of x, x^3, x^5, x^7, and x^9 are all even, the numerator of the probability is even, so D is not correct and only A remains. (Answer: A)

19. The more subsets there are, the smaller the common sum is and, since some subset must contain 15, the smallest possible common sum is 15. This can be done with $\{15\} \cup \{1, 14\} \cup \{2, 13\} \cup \cdots \cup \{7, 8\}$, so the largest number of common sum subsets is 8. (Answer: D)

20. To assure that no prime follows another, first arrange the 5 non-primes, then assign each of the 4 primes to one of the 6 spaces before, between, or beyond the non-primes. There are $5! \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 3(5!)^2$ ways to do this, and there are 9! arrangements in all, so the probability is $\frac{3(5!)^2}{9!} = \frac{5}{42}$. (Answer: C)