1. One die must be prime and the others 1; there are 3 choices for the prime (2, 3, or 5), and 3 choices for which die is prime, so 9 ways this can happen. There are 6^3 outcomes possible, so the probability is $9/6^3 = 1/24$. (Answer: B)

2. For some A and B, $6x^3+5x^2+Px+Q = (x^2+1)(Ax+B) = Ax^3+Bx^2+Ax+B$. Equate coefficients of like terms to find $P = A = 6$ and $Q = B = 5$, so $P + Q = 11$. (Answer: B)

3. Of the four choices, the only magical possibilities are 01/31/31 and 03/31/93; 02/29/58 is not possible since leap years are divisible by 4. (Answer: C)

4. Assume without loss of generality that $a > b > 0$. Since $a^2 - b^2 = (a-b)(a+b) = 91 = 7 \cdot 13$, either $a - b = 1$ and $a + b = 91$, or $a - b = 7$ and $a + b = 13$. Therefore, $(a, b) = (46, 45)$ or $(10, 3)$; since $46^2 + 45^2 > 1000$, $(a, b) \neq (46, 45)$ and $n = 10^2 + 3^2 = 109$. (Answer: E)

5. The x- and y-intercepts are $-b/m$ and b. Since $m = 36 - b$, $-b/m = -b/(36 - b) = b/(b - 36) = 1+36/(b-36)$, it must be that b and $36/(b-36)$ are integers. Therefore, each element of S corresponds to exactly one positive or negative factor of 36, and there are 18 of these. (Answer: E)

6. If x and y are the numbers of 2- and 3-axled vehicles, then $x + y = 120$ and $5x + 8y = 741$; solve this system to find $(x, y) = (73, 47)$, then calculate $6x + 10y = 6(73) + 10(47) = 908$. (Answer: B)

7. By cases. a is at most 12, which leads to $b^3 + c^2 = 2012 - 12^3 = 284$; the possibilities for b are $1, 2, \ldots, 6$, but $284 - b^3$ is not a square for any of these, so $a \neq 12$. If $a = 11$, then $b^3 + c^2 = 681$, so b is at most 8, which leads to $c^2 = 169 = 13^2$, so $a + b + c = 11 + 8 + 13 = 32$. (Answer: C)

8. Given $T + (D + H)/2 = 5$ and $H + (T + D)/2 = 7$, subtract and simplify to find $H = T + 4$. Substitute into the first equation and simplify to obtain $3T + D = 6$. Since $T, D > 0$ are integers, $T = 1, D = 3, H = 5$, and the total number of children is 9. (Answer: C)

9. Let $C_{i,j}$ = the value in row i, column j. Following the rules, determine by process of elimination that $C_{1,5} = 3, C_{1,4} = 4, C_{2,5} = 1, C_{3,5} = 2, C_{2,4} = 3$, and $C_{3,4} = 1$. (Answer: A)

10. $aa = a \cdot 11$, so the product with b is $a \cdot b \cdot 11 = (a \cdot b) \cdot 10 + (a \cdot b)$. For this to equal cba, the ones digit of $a \cdot b$ must be a; the possibilities for this in this problem are $(a, b) = (2, 6), (4, 6), (5, 3), (5, 7), (5, 9)$, and $(8, 6)$, with corresponding products $a \cdot b \cdot 11 = 132, 264, 165, 385, 495, 528$. These are all 3-digit numbers with distinct digits, but the only ones with 2nd digit equal to b are $264 = 44 \cdot 6$ and $495 = 55 \cdot 9$, so the answer is their sum, 759. (Answer: C)

11. Let r and r^2 be the solutions, so $x^2 - \frac{10}{9}x + c = (x - r)(x - r^2) = x^2 - (r + r^2)x + r^3$ and, equating coefficients, observe that $c = r^3$ and $r^2 + r = \frac{10}{9}$. This last equation has solutions $r = -5/3, 2/3$, but $r^3 = c > 0$, so $r = 2/3$ and $c = 8/27$ and $m + n = 35$. (Answer: D)

12. The equation simplifies to $xy = 0$, so the solution set is the union of the x- and y-axes. (Answer: C)

13. Subtract Sue's equation, $ac + b = 59$, from Thai's equation, $(a + b)c = 80$, to obtain $b(c - 1) = 21$, so the possibilities for b and c are $(b, c) = (1, 22), (3, 8), (7, 4)$, and $(21, 2)$. From Sue's (or Thai's) equation, reduce the possibilities to $(a, b, c) = (7, 3, 8), (13, 7, 4)$, and $(19, 21, 2)$; only the first of these gives the correct answer, $ab + c = 29$, so the sum is 18. (Answer: D)

14. R consists of the points in the first quadrant that are outside the circle C with diameter $\overline{AB}$ and between the lines which are tangent to C at A and B. The total area between the lines and in the first quadrant is 150, the total area of the circle is $25\pi \approx 78.54$, and the area of the portion of the circle below the x-axis is $25 \arcsin(3/5) - 12 \approx 4.09$, so R has area $\approx 150 - 78.54 + 4.09 = 75.55 \approx 76$, which was not an option. (The exam writers likely forgot to add back the piece of circle of area 4.09, so they thought the answer was 71.46, which rounds to 71, which was C.) (Answer: correct for all)

15. In a coordinate system with $A = (0,0), B = (4,0)$, and $C = (2,6)$, show that $D = (0,4), E = (6,2)$, the line $\overline{BC}$ has equation $y = -3x + 12$, and the line $\overline{AE}$ has equation $y = x/3$. Lines $\overline{BC}$ and $\overline{DE}$ intersect (along with the line $y = x$) at $F = (3,3)$, and $\overline{AE}$ intersects $\overline{BC}$ at $G = (3.6, 1.2)$. The area of overlap is twice the area of $\triangle AFG$. Since $\overline{AE}$ is perpendicular to $\overline{BC}$, the area is $2(1/2)\sqrt{(0.6)^2 + (1.8)^2}\sqrt{(3.6)^2 + (1.2)^2} = 7.2$ (Answer: B)

16. The integers $n = 1, 2, 3, 4, 5$ satisfy the conditions since they are the same in both bases. If $n = [ab]_6$ is a two digit number in base 6, where $1 \le a, b \le 5$, the conditions mean $6a + b = a + 9b \implies 5a = 8b$, which is impossible, so there are no such 2-digit numbers in base 6.

If $n = [abc]_6$ is a three digit number in base 6, where $0 \le a, b, c \le 5$ and $a, c \ne 0$, the conditions mean $36a + 6b + c = a + 9b + 81c \implies 3b = 5(7a - 16c)$, so $b = 0, 5$; $b \ne 0$ since $7a = 16c$ has no valid solution, so $b = 5 \implies 7a = 16c + 3$. Examine the cases $c = 1, 2, 3, 4, 5$, to find the only three digit solution, $(a, b, c) = (5, 5, 2)$, i.e., $n = 212$.

That makes 6 integers so far with the given property; in fact, there are no others, but this is more difficult to prove. To summarize: A number with d digits in base 6 is less than 6^d, and a number with d digits in base 9 is at least 9^{d-1}, so a number with the given property with d digits must satisfy $6^d > 9^{d-1} \implies d \le 5$, so it is enough to check the $d = 4$ and $d = 5$ cases. The $d = 4$ case reduces to showing that $215a + 27b - 75c - 728d = 0$ has no integer solutions with $0 \le a, b, c, d \le 5$ and $a, d \ne 0$; the $d = 5$ case reduces to showing that $1295a + 207b - 45c - 723d - 6560e = 0$ has no integer solutions with $0 \le a, b, c, d, e \le 5$ and $a, e \ne 0$. Have fun! (Answer: B)

17. There is a rational solution if and only if the polynomial factors over the integers; if p and r are prime, this is only possible if $px^2 + qx + r = (px + r)(x + 1)$ and $q = p + r$, or if $px^2 + qx + r = (px + 1)(x + r)$ and $q = pr + 1$. Either way, if p and r are odd primes, then $q > 2$ is even, so not prime; therefore, if (p, q, r) is in S, at least one of p and r is 2, which means every triple in S is of the form $(2, m + 2, m)$ or $(m, m + 2, 2)$ with m and $m + 2$ prime, or of the form $(2, 2m + 1, m)$ or $(m, 2m + 1, 2)$ with m and $2m + 1$ prime. 2 occurs in many such triples (most likely infinitely many, but this has not been proved – look up twin primes and Sophie Germain primes), for example $(2,5,2), (2,5,3), (3,5,2), (2,7,3), (3,7,2)(2,7,5), (5,7,2), (2,11,5)$, and $(5,11,2)$. These examples show that 2 and 5 occur in at least seven triples in S. On the other hand, for any odd prime n, the only triples in S that might contain it are $(2, n+2, n), (n, n+2, 2), (2, 2n+1, n), (n, 2n+1, 2), (2, n, n-2), (n-2, n, 2), (2, n, \frac{n-1}{2})$, and $(\frac{n-1}{2}, n, 2)$, but only four of these are in S when $n = 3$, and for $n > 5$, at most six of these can be in S since $n - 2, n$, and $n + 2$ are all prime only when $n = 5$. So 2 and 5 are the only primes that appear in S at least seven times. (Answer: C)

18. You can use identities, but the easiest way is to use a calculator to find $\frac{m}{n^2} = \left[\tan\left(\frac{1}{4}\arccos(7/18)\right)\right]^2 = 0.090909\cdots = \frac{1}{11}$, so $\frac{\sqrt{m}}{n} = \frac{1}{\sqrt{11}} = \frac{\sqrt{11}}{11}$, so $m + n = 11 + 11 = 22$. (Answer: B)

19. There are $C_{12,6} = 924$ committees in all; of these, there are $C_{9,6} = 84$ which have NO freshmen, 84 which have no sophomores, etc., there is only 1 which has NO freshmen or sophomores, 1 which has no freshmen or juniors, etc., only 1 which has no ENGR students, and only 1 which has no CS students. By the inclusion-exclusion principle, the number of committee which has at least one member of each class and major is $924 - (4 \cdot 84 + 2 \cdot 1) + 6 \cdot 1 = 592$. (Answer: D)

20. Subtract $10xy$ from both sides and complete the square $x^2 - 10xy = (x - 5y)^2 - 25y^2$ to obtain $(x-5y)^2 - 36y^2 + 23 = 0$. Factor the difference of squares and rearrange to obtain $(11y - x)(x + y) = 23$. Since the solutions are integers and 23 is prime, the possibilities for the factors (not x and y) are $\pm(1, 23)$ and $\pm(23, 1)$. Each of these results in a system of equations with a unique solution; for example, $(1, 23) \implies 11y - x = 1$ and $x + y = 23 \implies (x, y) = (21, 2)$. The other solutions are $(-21, -2)$ and $\pm(-1, 2)$, so the answer is $21 + 21 + 1 + 1 = 44$. (Answer: C)