1. A total of 50 problems, minus 12 problems in common, makes 38 distinct problems in all. (Answer: D)

2. The third side of a triangle must be longer than the difference of the other two sides and shorter than their sum. Therefore if c is the length of the third side: $8.1 - 1.4 < c < 8.1 + 1.4 \implies 6.7 < c < 9.5$. Of the choices provided, 8 is the only number that falls into this range. (Answer: D)

3. The first equation minus the second is $(3e)x + (3e)y = 3e \implies x + y = 1 \implies x = 1 - y$. Substitute for x in the first equation to get $y = 2 \implies x = -1 \implies b - a = y - x = 3$. (Answer: E)

4. Just factor the numbers given: $2014 = 2 \cdot 19 \cdot 53 \implies \{1, 18, 52\}$, $2015 = 5 \cdot 13 \cdot 31 \implies \{4, 12, 30\}$, and $2016 = 2^5 \cdot 3^2 \cdot 7 \implies \{1, 2, 6\}$, which has the desired property. (Answer: C)

5. The lines intersect at some point $(x, 0)$. Set $y = 0$ in each equation to find $x = -b/2$ and $x = 6/m$, respectively. These ae the same point, so $-b/2 = 6/m \implies mb = -12$. (Answer: B)

6. Play around a bit, starting with $n = 3$, and hopefully find $\frac{1}{4} = \frac{1}{20} + \frac{1}{5} = \frac{1}{12} + \frac{1}{6} = \frac{1}{8} + \frac{1}{8}$. In general, if $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{a+b}{ab} = \dfrac{1}{n} = \dfrac{k}{kn}$ with k *as small as possible*, then $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{kn} + \dfrac{k-1}{kn}$, so $k-1$ is a factor of n, and the pairs (a, b) with $a \geq b > 0$ and $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{n}$ are of the form $(a, b) = (kn, kn/(k-1))$, where $k - 1$ is a factor of n: $k = 2$ is the smallest possible, corresponding to $\frac{1}{2n} + \frac{1}{2n} = \frac{1}{n}$, and $k = n + 1$ is the largest possible, corresponding to $\frac{1}{n(n+1)} + \frac{1}{n+1} = \frac{1}{n}$. Therefore, the number of solutions is the number of factors of n, and the smallest n with 3 factors is $n = 4$. (Answer: B)

7. The possibilities for b are fewest, so with a calculator, store the values $5, 10, \ldots$ for B, and use the TABLE feature with formula $Y = \sqrt{2013 - B^2 - X^3}$ to find integer pairs $(a, c) = (Y, X)$. The solution $(a, b, c) = (4, 10, 43)$ is quickly found this way, so $a + b + c = 57$. (Answer: B)

8. $A = 11$, since otherwise two different letters are both 11 or some letter is $\geq 33 > 27$. From $MTYC = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7$, similar considerations demand that some letter is $3 \cdot 5 = 15$ and the others are 3, 5, and 7, so $M + T + Y + C = 3 + 5 + 7 + 15 = 30$. (Answer: A)

9. As sets of values, $\{P(0), P(3)\} = \{1, 139\}$ and $\{P(1), P(2)\} = \{1, 689\}$ or $\{13, 53\}$. The coefficients of P are non-negative, so P is increasing on $[0, \infty)$, and the values must be $P(0) = 1, P(1) = 13, P(2) = 53, P(3) = 139$. One way to continue is to set $P(x) = ax^3 + bx^2 + cx + d$, use the above values to write the equations $P(0) = d = 1, P(1) = a + b + c + d = 13, P(2) = 8a + 4b + 2c + d = 53$, and $P(3) = 27a + 9b + 3c + d = 139$, and solve these to find $(a, b, c, d) = (3, 5, 4, 1) \implies P(-1) = -a + b - c + d = -3 + 5 - 4 + 1 = -1$. Alternatively, if you know that the k^{th} differences of a k^{th}-degree polynomial are constant, you can use this fact to quickly find the same result. (Answer: B)

10. Let $\cos_{RAD}(x)$ be the cosine function which takes a radian argument, and let $\cos_{DEG}(x)$ be the cosine function which takes a degree argument. The relation between these is $\cos_{DEG}(x) = \cos_{RAD}(\pi x/180)$, so the problem is to find the smallest positive solution to $\cos_{RAD}(x) = \cos_{DEG}(x) \iff \cos_{RAD}(x) = \cos_{RAD}(\pi x/180)$. With a graphing calculator (in radian mode), it is easy to find that the first positive intersection of the curves $Y_1 = \cos(X)$ and $Y_2 = \cos(\pi X/180)$ occurs at approximately $(6.1754042, 0.99419723)$. (Answer: 6.175)

11. By the Pythagorean theorem, $BD = 10$, so $\triangle ABD$ is isoceles with base $AB = 6$ and sides $BD = DA = 10$. Let $2\alpha = \angle A$; by the law of sines, $\frac{BE}{\sin \alpha} = \frac{6}{\sin(\pi - 3\alpha)} = \frac{6}{\sin 3\alpha}$ and $\frac{10 - BE}{\sin \alpha} = \frac{10}{\sin 3\alpha}$. It follows that $\frac{\sin \alpha}{\sin 3\alpha} = \frac{BE}{6} = \frac{10 - BE}{10} \implies 10BE = 60 - 6BE \implies BE = \frac{15}{4}$ (Answer: A)

12. There are two possibilites each for L and M, so 4 possible points of intersection: $(a, b) = (0, 4), (4, 0), (-4, 12)$ or $(12, -4) \implies 3a + b = 4, 12, 0$, or 32, so only 8 is not possible. (Answer: C)

13. If n = length of the first trip and k = number of trips, then $n+(n+2)+(n+4)+\cdots+(n+2(k-1)) = 366 \implies kn+2(1+2+\cdots+(k-1)) = k(n+k-1) = 366 = 2\cdot 3\cdot 61$. k must be a factor of 366, so the positive integer solutions are $(k,n) = (1,366),(2,182),(3,120)$, and $(6,56)$; since $n \le 90$, only the last of these works, and the trips were of lengths 56, 58, 60, 62, 64, and 66. (Answer: B)

14. Most likely, the problem should have been: "For a 6-digit bit string s, let $R(s)$ be the reverse of s and let $O(s) = 111111 - s$ be the opposite of s; e.g., $R(110101) = 101011$ and $O(110101) = 001010$. Find the largest possible size of a set S of 6-digit bit strings, such that $s \in S \implies R(s), O(s) \notin S$." Each of the $2^6 = 64$ strings is either a palindrome, with $R(s) = s$; a palopposite, with $R(s) = O(s)$; or neither. There are $2^3 = 8$ palindromes, and none may be in S. There are $2^3 = 8$ palopposites which form pairs such as $\{011001, 100110\}$, and at most one from each of these 4 pairs may be in S. The remaining 48 strings fall into 12 quartets of the form $\{s, R(s), O(s), R(O(s)) = O(R(s))\}$; at most 2 from this quartet may be in S, either $\{s, O(R(s))\}$ or $\{R(s), O(s)\}$. Thus, S contains at most $4 + 2(12) = 28$ strings. S is not unique – there are 2^{16} such sets! Writing strings as decimal numbers, one example is $S = \{7,11,21,25,1,31,2,47,3,15,4,55,5,23,6,39,9,27,10,43,13,19,14,35,17,29,22,37\}$. So the correct answer to the likely problem is 28, which was not an option. (Answer: Correct for all students)

15. The non-intersecting "diagonals" PR and QS lie on perpendicular lines (which intersect at T), so the area is $\frac{1}{2}|PR||QS|$. $\triangle QTS \cong \triangle PTR$, so $|QS| = |PR| = 8\sqrt{2}$, so the area is exactly 64. (Answer: E)

16. In other words, find the smallest pair (a,b) with $a^2 = 2b^2+2$ and $a > 10$. Use the TABLE function on a calculator with $Y = \sqrt{2X^2+2}$ to quickly find the pair $(a,b) = (58,41)$, so $a - b = 17$. (Answer: C)

17. Just write out the possibilities to find 2 such numbers that begin with 1 (13524, 14253), 3 that begin with 2 (24135, 24153, 25314), and 4 that begin with 3 (31425, 31524, 35241, 35142); by symmetry, there are 3 that begin with 4 and 2 that begin with 5, so 14 such numbers with no consecutive digits. There are 5! 5-digit numbers with distinct digits, so the probability is $14/5! = 7/60$. (Answer: B)

18. The region is the union of a quarter-circle C_4 of radius 4 in the first quadrant, a quarter-circle C_3 of radius 3 in the second quadrant, and the triangle T with vertices $O(0,0), P(-3,0), Q(0,4)$. Estimate the area inside T but outside C_3 by a right triangle with height 1 and base 3/4, to find $A > \frac{\pi}{4}(3^2+4^2) + \frac{1}{2}(1)(\frac{3}{4}) \approx 20.009954$; the neglected area is contained in the right triangle with vertices $(0,3), (0,\sqrt{8})$, and $(-1,\sqrt{8})$, which has area $(3-\sqrt{8})/2 \approx 0.0858$, so $20.009 < A < 20.096$, so only B works. Alternatively, solve $y = \frac{4}{3}x + 4$ and $x^2 + y^2 = 9$ to find that T and C_3 intersect at $R(-21/25, 72/25)$. The area of $\triangle OQR$ is 42/25 and the area of the remaining sector of C_3 is $\frac{1}{2}3^2\arctan(72/21)$, so the exact area is $4\pi + 4.5\arctan(24/7) + 42/25 \approx 20.03788 \approx 20.04$. (Answer: B)

19. By the quadratic formula, these polynomials factor iff $m^2 - 4n$ and $m^2 + 4n$ are perfect squares. If your calculator can deal with two-variable tables, look for integer values of $\sqrt{m^2-4n} + \sqrt{m^2+4n}$, $1 \le m,n \le 99$. Otherwise, suppose $m^2 - 4n = (m-k)^2$ and $m^2 + 4n = (m+j)^2$ for some integers $j,k > 0$; it follows that $2mj + j^2 = 4n = 2mk - k^2 \implies 2m(k-j) = j^2 + k^2 \implies j,k$ are both odd or both even $\implies$ both sides are divisible by 4 $\implies j = 2p$ and $k = 2q$ for some $p,q \ge 1 \implies m(q-p) = p^2+q^2 \implies q = p+d$ for some $d \ge 1 \implies m = \frac{2p^2}{d} + 2p + d$ and $n = (2mj+j^2)/4 = p(m+p)$. Plug in values of p and d for which $d|2p^2$ and record those pairs with $m,n < 100$; since $m > 2p \implies n > 3p^2$, it is only necessary to check through $p = 5$ and, writing (m,n) instead of (n,m) as on the test, find the 7 pairs $(m,n) = (5,6),(13,30),(10,24),(25,84),(17,60),(15,54)$, and $(20,96)$. (Answer: D)

20. The triangles have right angles at A and B, so $BC = \sqrt{50^2-40^2} = 30$, $area(\triangle BCD) = \frac{1}{2}(30)(40) = 600$, $AC = \sqrt{50^2-14^2} = 48$, and $area(\triangle ACD) = \frac{1}{2}(14)(48) = 336$. In coordinates with C at the origin, D at $(50,0)$, and A to the right of B, CB is on the line $y = 7x/24$ and BD is on the line $y = 3(50-x)/4$, so the lines intersect at $E = (36, 21/2)$. Therefore, $area(\triangle ACD \cup \triangle BCD) = area(\triangle ACD) + area(\triangle BCD) - area(\triangle ECD) = 600 + 336 - \frac{1}{2}(50)(21/2) = 673.5$. (Answer: D)