

1. When each is given in Fahrenheit, the sum of the high and low temperatures for Memphis, TN on a particular day is 68. What is the sum of the high and low temperatures in Memphis on the same day if each is given in Celsius? *Hint: $F = (9/5)C + 32$*
 A. 20/9 B. 36/5 C. 20 D. 772/9 E. 772/5

2. Four rings of different sizes are stacked on a post, in ascending order (smallest on top). There are two other empty posts. You are able to move one ring at a time (taking the top ring from one post and moving it to another post), but you may never place a larger ring on a smaller ring. What is the minimum number of moves required to move the entire stack to the middle post?



A. 12 B. 14 C. 15 D. 16 E. 17

3. Find the sum of all of the real solutions to $\left|4 - \left|3 - \left|2 - \left|1 - x\right|\right|\right|\right| = 0$.

A. -2 B. -1 C. 0 D. 1 E. 2

4. A regular pentagon is rotated 36° around its center to produce a second pentagon. The area of the intersection of the two pentagons is what fraction of the area of the original pentagon, to the nearest whole percent?

A. 88% B. 89% C. 90% D. 91% E. 92%

5. The area of the four-sided region in the first quadrant bounded by the x -axis, y -axis, and the lines $3x + 4y = 12$ and $2y - x = 2$ is cut in half by the line $y = kx$. Find k .

A. 33/76 B. 2/5 C. 11/19 D. 1/2 E. 21/38

6. Each letter in the equation $\sqrt{AMATYC} = MYM$ represents a distinct non-zero decimal digit. Find T .

A. 3 B. 4 C. 5 D. 6 E. 7

7. The line $y = mx + b$ is tangent to the circle $(x + 1)^2 + (y - 1)^2 = 25$ at $(3, 4)$. Find $m + b$.

A. 5/12 B. 5/2 C. 7/2 D. 20/3 E. 35/4

8. Compute the following (where $i = \sqrt{-1}$): $\sum_{n=1}^{2018} (i^n + i^{-n})$

A. 2 B. -2 C. 0 D. $i - 2$ E. $2 - i$

9. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says "Z is a Knight.", Y says "Z is not a Knight.", and Z says "I am a Knave." Which of the following correctly identifies all three people?

A.	B.	C.	D.	E.
X is the spy.	X is the spy.	X is the knight.	X is the knight.	X is the knave.
Y is the knight.	Y is the knave.	Y is the knave.	Y is the spy.	Y is the knight.
Z is the knave.	Z is the knight.	Z is the spy.	Z is the knave.	Z is the spy.

10. Suppose $\log_A B = \log_B A$ and $A > B > 1$. Which of the following is true?

A. $AB = 1$ B. $AB = e$ C. $AB = e^2$ D. $AB = 10$ E. This is not possible.

11. Let M be the greatest integer less than 30 such that $M!(M+1)!/2$ is a perfect square. Let N be the greatest integer that divides $c^4 - c^2$ for all integers $c > 1$. Find $M + N$.
- A. 19 B. 21 C. 26 D. 29 E. 36
12. A store carries 11 different types of bagels. Late one day, they had only 3 onion bagels left and 5 plain bagels left, but still had several dozen of each of the other types. If someone wanted to purchase a dozen bagels then, how many different combinations of 12 bagels would be possible (assuming that bagels of the same type are indistinguishable)?
- A. 293,710 B. 594,880 C. 594,946 D. 646,184 E. 646,646
13. Kara repeatedly flips a fair coin (the probability of flipping heads and the probability of flipping tails are both $\frac{1}{2}$), and stops when she flips two consecutive heads. What is the expected number of flips?
- A. 3 B. 4 C. 5 D. 6 E. 7
14. S is a set of four distinct real numbers, with greatest element z . If one adds each possible pair of elements of S , the results are (in ascending order): 2, 3, 4, 5, x , y . Find the sum of all possible values of z .
- A. $17/2$ B. $26/3$ C. 9 D. $28/3$ E. 10
15. How many of the following are both a well-defined function on \mathbb{R} and also equivalent to the identity function $f(x) = x$ on \mathbb{R} ? $g(x) = \ln e^x$, $h(x) = e^{\ln(x)}$, $k(x) = \sqrt{x^2}$, $m(x) = \sqrt[3]{x^3}$, $n(x) = 1 + \frac{x^2-1}{x+1}$, $p(x) = \sin(\arcsin x)$, $q(x) = \arctan(\tan x)$, $r(x) = \pm|x|$, $s(x) = (0.\bar{9})x$
- A. 1 B. 2 C. 3 D. 4 E. 5
16. The graph of the equation $y = x^2 - 2kx + k$ is a parabola for any real number k . If the vertex of this parabola occurs at the point (a, b) , find the greatest possible value of b .
- A. 0 B. $1/8$ C. $1/4$ D. $1/2$ E. 1
17. The three digit decimal number abc is equivalent to the three digit number cba in hexadecimal (base 16). Find $a + b + c$.
- A. 9 B. 11 C. 13 D. 15 E. 17
18. A collection of identical spheres can be formed into a "square" pyramid (a pyramid with a base (bottom layer) made up of $n \times n$ spheres whose next layer is made up of $(n-1) \times (n-1)$ spheres, continuing this way up to the top layer of 1 sphere). The same collection of spheres can also be formed into a single-layer $k \times k$ "square" where $k < 100$. Find the largest possible value of k for such a collection of spheres and record it on your answer sheet.
19. Consider a set of positive integers less than 100 such that no two elements have a sum of 100. Let M be the maximum number of distinct elements that such a set can contain. Let C be the coefficient of x^5 in the expansion of $(x - 0.5)^8$. Find $|C - M|$.
- A. 42 B. 43 C. 44 D. 56 E. 57
20. Suppose that $f(x) = \frac{x^3 + x^2 + cx + d}{x+2}$ is equivalent to $g(x) = ax^2 + bx + 4$ on its domain. Find $f(3)$.
- A. $38/5$ B. 9 C. 10 D. $54/5$ E. 12

Test #2

AMATYC Student Mathematics League

Winter/Spring 2018

1. A
2. C
3. E
4. B
5. A
6. E
7. D
8. B
9. E
10. E
11. D
12. C
13. D
14. A
15. C
16. C
17. B
18. 70
19. E
20. C