1. (8.2) Use Gauss-Jordan elimination to find the complete solution to the system.

$$\begin{cases} 2x - 8y + z = 3 \\ -2x + 5y - 5z = 1 \\ 2x - 3y + 9z = -5 \end{cases}$$

2. (8.3) Solve for a by evaluating the determinant.

$$\begin{vmatrix} -1 & a & 5 \\ 2 & 4 & 1 \\ 0 & 1 & -1 \end{vmatrix} = 21$$

3. (8.3) Use Cramer's Rule to solve the systems of equations. You may use your calculator to evaluate the determinants.

(a) $\begin{cases} 5x - 6y = 1 \\ -2x + 7y = -3 \end{cases}$

(b) $\begin{cases} 2x \quad - z = 14 \\ 3x - y + 5z = 0 \\ 4x + 2y + 3z = -2 \end{cases}$

4. (8.4) Let

$$A = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -2 & 1 \\ -5 & 1 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 0 & -2 \end{bmatrix}$$

Carry out the indicated operations if possible.

(a) A^2 **(b)** $AB - 3C$ **(c)** $BA - 3C$

5. (8.5) Find the partial fraction decomosition.

$$\frac{2x^3 - 2x^2 - 3x - 5}{(x+1)^2(x^2+2)}$$

6. (8.6) Solve the system of equations.

$$\begin{cases} 3x^2 + 2xy - 2y^2 = -1 \\ xy - y^2 = -2 \end{cases}$$

7. (8.7) Sketch the solution set of the system of linear inequalities.

$$\begin{cases} x^2 - 3x - y < 0 \\ -x + 3y \le -3 \end{cases}$$

8. (9.1) Write the first five terms of the following sequences:

(a) $a_n = \dfrac{(-1)^{n+1}}{2n-1}$

(b) $a_1 = 1$; $a_n = 1 + 2a_{n-1}$

9. (9.1) Find the nth term of the sequence.

$$\tfrac{1}{2}, -\tfrac{3}{5}, \tfrac{1}{2}, -\tfrac{7}{17}, \tfrac{9}{26}, -\tfrac{11}{37}, \dots$$

10. (9.1) Evaluate the sum by hand: $\displaystyle\sum_{k=1}^{5} (-1)^k k^3$

11. (9.2) For the sequence: $-13, -6, 1, 8, \dots$

(a) Find the 9th term.

(b) Find the nth term.

(c) Add up the first 100 terms.

12. (9.2) A brick staircase has a total of 25 steps. The bottom step requires 80 bricks and each successive step requires three fewer bricks than the previous step. How many bricks are required for the top step? How many bricks are required to build the entire staircase?

13. (9.3) For the sequence: $1, -3, 9, -27, \dots$

(a) Find the 9th term.

(b) Find the nth term.

(c) Add up the first 12 terms.

14. (9.3) A force is applied to a particle, which moves in a straight line, in such a way that after each second (after the initial move) the particle travels exactly two-thirds the distance that it moved in the preceding second. If this pattern continues indefinitely and the particle moved 12 cm in the initial move, what is the total distance the particle with travel?

15. (9.4) Use mathematical induction to prove that the formula is true for all natural numbers n:

$$1 + 4 + 7 + \cdots + (3n-2) = \frac{n(3n-1)}{2}$$

16. (9.5) Evaluate: $\binom{7}{2} + \binom{7}{4}$

17. (9.5) Expand: $(2x - y)^5$

18. (9.5) Find the term that contains a^{17} in the expansion of $(a + b)^{20}$.

Answers

1. $(2, 0, -1)$

2. 3

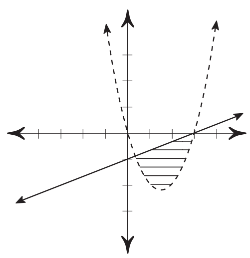
3. (a) $\left(-\frac{11}{23}, -\frac{13}{23}\right)$; (b) $(5, -5, -4)$

4. (a) $\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$; (b) $\begin{bmatrix} -22 & 3 & 2 \\ -1 & 5 & 1 \end{bmatrix}$

(c) not possible

5. $\dfrac{1}{x+1} - \dfrac{2}{(x+1)^2} + \dfrac{x-3}{x^2+2}$

6. $(1, 2)$, $(1, -1)$, $(-1, -2)$, $(-1, 1)$

7. 

8. (a) $\left\{1, -\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}\right\}$

(b) $\{1, 3, 7, 15, 31\}$

9. $a_n = \dfrac{(-1)^{n+1}(2n-1)}{n^2+1}$

10. -81

11. (a) 43

(b) $a_n = 7n - 20$

(c) 33,350

12. Eight bricks for the top step, 1,100 total bricks for the staircase.

13. (a) 6561

(b) $a_n = (-3)^{n-1}$

(c) $-132,860$

14. 36 cm

15. Base case: $1 = \frac{1(3(1)-1)}{2}$, is true; Hypothesis: $1 + 4 + 7 + \cdots + (3k-2) = \frac{k(3k-1)}{2}$;
Inductive step: $1+4+7+\cdots+(3k-2)+(3(k+1)-2) = \frac{k(3k-1)}{2} + 3k+1 = \frac{k(3k-1)+2(3k+1)}{2} = \frac{3k^2+5k+2}{2} = \frac{(k+1)(3k+2)}{2} = \frac{(k+1)(3(k+1)-1)}{2}$
Therefore the statement is true for all natural numbers.

16. 56

17. $32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

18. $1140a^{17}b^3$