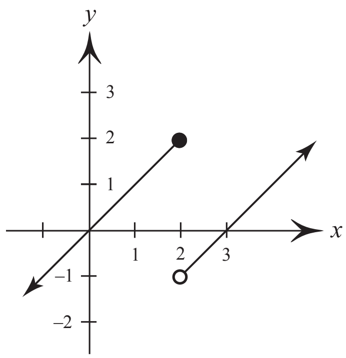
1. Use the graph of f below to answer the questions that follow.



(a) $\lim_{x\to 2^-} f(x)$ (c) $\lim_{x\to 2} f(x)$

(b) $\lim_{x\to 2^+} f(x)$ (d) What is $f(2)$?

2. Find the limit L, then use the δ-ε definition to prove that the limit is L.

$$\lim_{x\to 6}\left(\frac{x}{2}-5\right)$$

For exercises 3–6, find each limit (if it exists).

3. $\lim_{x\to -3}\sqrt{6-x}$ **4.** $\lim_{x\to -4}\frac{x^2+10x+24}{x+4}$

5. $\lim_{x\to 1}\frac{|x-1|}{x-1}$ **6.** $\lim_{\Delta x\to 0}\frac{\sqrt{9+\Delta x}-3}{\Delta x}$

For exercises 7–8, determine the intervals on which the function is continuous.

7. $\frac{x+7}{x-8}$ **8.** $f(x)=\begin{cases}(x-2)^2, & x<2\\ \sin\pi x, & x\geq 2\end{cases}$

For exercises 9–10, find each limit (if it exists).

9. $\lim_{x\to -3^-}\frac{x-1}{x+3}$ **10.** $\lim_{x\to\pi}\csc x$

11. Use the limit definition of the derivative to find $f'(x)$.

$$f(x)=\frac{1}{x^2+2}$$

For exercises 12–17, find the derivative of each function.

12. $f(x)=\frac{3}{\sqrt{x}}$ **13.** $g(x)=x^2e^{2x}$

14. $h(t)=\ln\sqrt{4t^3+7}$ **15.** $r(\theta)=\frac{1+\sin\theta}{1-\tan\theta}$

16. $f(x)=\arctan x^2$ **17.** $p(x)=5^x$

18. Find the equation of the tangent line to the graph of f at the given point.

$$f(x)=\frac{2x^3+3}{x^2},\quad (1,5)$$

19. Use implicit differentiation to find dy/dx

$$x\sin y-4=x^2+\cos y$$

20. The edges of a cube are expanding at a rate of 8 cm per second. How fast is the surface area changing when each edge is 6 cm?

21. Find the absolute extrema of f on the given closed interval.

$$f(x)=x^2+5x,\quad [-4,0]$$

22. Find the value c guaranteed by the Mean Value Theorem.

$$f(x)=x^{2/3},\quad [1,8]$$

23. Find the intervals on which f is increasing or decreasing, then determine all relative extrema.

$$f(x)=\sqrt{x}(x-3)$$

24. Find the points of inflection and discuss the concavity of the function.

$$g(x)=x^3-9x^2$$

25. Demonstrate the Second Derivative Test while finding all relative extrema.

$$f(x) = 2x \ln x$$

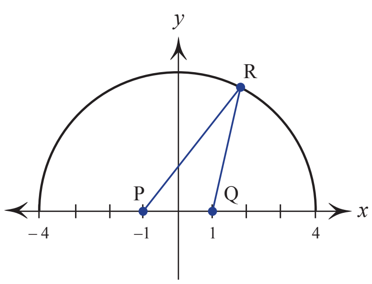
26. Find the limit if it exists.

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+1}}{-2x}$$

For exercises 27–28, evaluate the limit using L'Hôpital's Rule.

27. $\lim_{x \to \infty} x^2 e^{-x}$ **28.** $\lim_{x \to \pi/4} \frac{1 - \tan x}{4x - \pi}$

29. Point R is any point on the semicircle $y = \sqrt{16 - x^2}$. If P is the point $(-1, 0)$ and Q is $(1, 0)$, find the largest possible value for PR + RQ.



30. The diameter of a sphere is 12 cm, with a maximum possible error of ± 0.05 cm. Use differentials to approximate the possible propagated error in calculating the surface area of the sphere.

For exercises 31–36, evaluate each indefinite integral.

31. $\int \frac{x^4+8}{x^3}\, dx$ **32.** $\int \frac{2}{\sqrt{5x}}\, dx$

33. $\int (\sin x - \csc^2 x)\, dx$ **34.** $\int x^2 e^{5x^3}\, dx$

35. $\int \frac{x^2 - 7x + 14}{x - 4}\, dx$ **36.** $\int \frac{\sin x}{\sqrt{\cos x}}\, dx$

37. An airplane taking off from a runway travels 3600 feet before lifting off. The airplane starts from rest, moves with constant acceleration, and makes the run in 30 seconds. At what speed is it traveling when it lifts off?

38. Sketch the region whose area is given by the definite integral, then use geometry to evaluate the integral.

$$\int_0^{10} (5 - |x - 5|)\, dx$$

39. Use the Riemann Sum to evaluate the definite integral.

$$\int_{-1}^{3} (x^2 + 5)\, dx$$

For exercises 40–41, evaluate the definite integral.

40. $\int_0^{\pi/4} \sin x\, dx$ **41.** $\int_1^e \frac{\ln \sqrt{x}}{x}\, dx$

42. Find the area bounded by the graphs of the equations:

$$y = \sec^2 x,\ y = 0,\ x = 0,\ x = \frac{\pi}{3}$$

43. Use the Fundamental Theorem of Calculus to find $F'(x)$.

$$F(x) = \int_1^x \sqrt{1 + t^5}\, dt$$

Answers

1. (a) 2; (b) −1; (c) dne; (d) 2

2. $L = -2$; For every $\varepsilon > 0$, if $|x-6| < \delta = 2\varepsilon$ then $|\frac{x}{2} - 5 - (-2)| = |\frac{x}{2} - 3| = \frac{1}{2}|x-6| < \frac{1}{2} \cdot 2\varepsilon = \varepsilon$

3. 3 4. 2 5. dne 6. $\dfrac{1}{6}$

7. $(-\infty, 8) \cup (8, \infty)$ 8. $(-\infty, \infty)$

9. ∞ 10. dne

11. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} =$
$\lim\limits_{h\to 0} \dfrac{1/((x+h)^2+2) - 1/(x^2+2)}{h} =$
$\lim\limits_{h\to 0} \dfrac{x^2+2-((x+h)^2+2)}{h(x^2+2)((x+h)^2+2)} =$
$\lim\limits_{h\to 0} \dfrac{x^2+2-x^2-2hx-h^2-2}{h(x^2+2)((x+h)^2+2)} =$
$\lim\limits_{h\to 0} \dfrac{-2hx-h^2}{h(x^2+2)((x+h)^2+2)} =$
$\lim\limits_{h\to 0} \dfrac{-2x-h}{(x^2+2)((x+h)^2+2)} = \dfrac{-2x}{(x^2+2)^2}$

12. $-\dfrac{3}{2\sqrt{x^3}}$

13. $2xe^{2x}(1+x)$ 14. $\dfrac{6t^2}{4t^3+7}$

15. $\dfrac{\cos\theta - \sin\theta + \sec^2\theta + \sec^2\theta\sin\theta}{(1-\tan\theta)^2}$

16. $\dfrac{2x}{1+x^4}$ 17. $5^x \ln 5$ 18. $y = -4x + 9$

19. $\dfrac{2x - \sin y}{x\cos y + \sin y}$ 20. 576 cm^2/sec

21. Abs max: $(0,0)$; Abs min: $\left(-\frac{5}{2}, -\frac{25}{4}\right)$

22. $f'\left(\frac{2744}{729}\right) = \frac{3}{7}$

23. decreasing on $(0,1)$; increasing on $(1,\infty)$; relative min at $(1,-2)$

24. inflection point: $(3,-54)$; concave down on $(-\infty,3)$; concave up on $(3,\infty)$

25. $f''(e^{-1}) = 2e > 0 \therefore$ rel min at $\left(\frac{1}{e}, -\frac{2}{e}\right)$

26. $\dfrac{1}{2}$ 27. 0 28. $-\dfrac{1}{2}$ 29. $2\sqrt{17}$

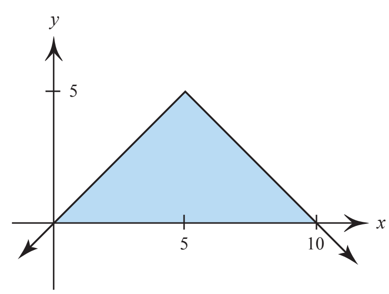
30. $\pm\dfrac{6\pi}{5}$ cm^2 31. $\dfrac{x^2}{2} - \dfrac{4}{x^2} + C$

32. $\dfrac{4\sqrt{5x}}{5} + C$ 33. $-\cos x + \cot x + C$

34. $\dfrac{1}{15}e^{5x^3} + C$ 35. $\dfrac{x^2}{2} - 3x + \ln(x-4)^2 + C$

36. $-2\sqrt{\cos x} + C$ 37. 240 ft/sec

38. $A = 25$ un^2



39. $\dfrac{88}{3}$ 40. $1 - \dfrac{\sqrt{2}}{2}$ 41. $\dfrac{1}{4}$

42. $\sqrt{3}$ un^2 43. $\sqrt{1+x^5}$