CONTENT This exam will cover the material discussed in the Trig Review (Appendix D), sections 1.4, 1.5, all of chapter 2.

TOPICS You should be comfortable with the following topics:

Limits, evaluating a limit analytically, evaluating a limit numerically, ε-δ definition of the limit, indeterminate form of a limit, the Squeeze Theorem, continuity, removable discontinuity, one-sided limits, $f(x) = [\![x]\!]$ (the greatest integer function), Intermediate Value Theorem, vertical asymptotes, derivative, limit definition of the derivative, alternate form of the limit definition of the derivative, non-differentiability, the graph of the derivative.

FORMULAS You should have the following formulas memorized.

Limit Definition of the Derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Alternate Form of the Derivative

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

PRACTICE PROBLEMS

1. (App D) One of the acute angles in a right triangle measures 25°. If the hypotenuse is 6 feet long, find the length of the side opposite the 25° angle. Round your answer to 3 decimal places.

2. (App D) For each equation, find *all* solutions for θ such that $0 \le \theta < 2\pi$.

(a) $\sec\theta = \dfrac{2\sqrt{3}}{3}$ **(b)** $2\sin^2\theta - \sin\theta = 1$

3. (1.4) Use the Law of Exponents to simplify each expression.

(a) $\dfrac{(6x^2)^3}{4x^7}$ **(b)** $\dfrac{\sqrt{y\sqrt[3]{y}}}{\sqrt{xy}}$

4. (1.4) Find the function in the form $f(x) = Cb^x$ whose graph passes through the points $(-1, 8)$ and $\left(1, \frac{1}{2}\right)$.

5. (1.5) Using the functions $f(x) = 3x + 1$ and $g(x) = 8 + \sqrt{x}$, find the following.

(a) $(g \circ f)(16)$

(b) $g^{-1}(x)$, and its domain

(c) $(f^{-1} \circ g^{-1})(24)$

6. (1.5) Write the expression as a single logarithm.

(a) $\frac{1}{2}\ln a - (\ln b + 2\ln c)$

(b) $5\ln x - \ln y + 3\ln z$

7. (1.5) Solve. Round your answer to three decimal places.

(a) $3^x = \dfrac{1}{9}$ **(b)** $\ln e^{2u} = 10$

(c) $1.2e^t = 700$

8. (2.2) Find the value of the limit numerically by creating a table using x-values: $-2.1, -2.01, -2.001, -1.999, -1.99, -1.9$.

$$\lim_{x\to -2} \left(\frac{\ln(x^2+1) - \ln(5)}{x+2} \right)$$

9. (2.2) Use the graph of f to answer the following:

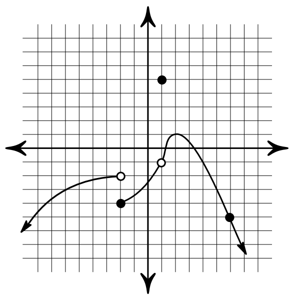
(a) $\lim_{x\to -2^-} f(x)$

(b) $\lim_{x\to -2^+} f(x)$

(c) $\lim_{x\to -2} f(x)$

(d) $\lim_{x\to 1} f(x)$

(e) $f(1)$



10. (2.2) Find the limit.

(a) $\lim_{x\to 2^+} \frac{x}{4-x^2}$ **(b)** $\lim_{x\to 4} \frac{x+4}{x^2-16}$

(c) $\lim_{x\to 2} \frac{1}{|x-2|}$ **(d)** $\lim_{\theta\to \pi/2^-} \tan\theta$

11. (2.3) Find the limit.

(a) $\lim_{x\to 2} \frac{2}{x+2}$ **(b)** $\lim_{x\to -3} \frac{x^2-9}{x+3}$

(c) $\lim_{\Delta x\to 0} \frac{2(x+\Delta x)^2 - 2x^2}{\Delta x}$

(d) $\lim_{h\to 0} \frac{\sqrt{h+1}-1}{h}$

12. (2.4) Find the limit L. Then use the ε-δ definition to prove that the limit is L.

$$\lim_{x\to 5} x^2 + 2$$

13. (2.5) Determine the intervals on which the functions are continuous.

(a) $g(x) = \frac{x^2 - 14x + 49}{x^2 - 49}$

(b) $r(x) = \begin{cases} x^2 - 3x & x < 3 \\ \sqrt{x-3} & x \geq 3 \end{cases}$

14. (2.5) For $g(x) = 2[\![x]\!] - 1$, find each of the following limits.

(a) $\lim_{x\to 3^-} g(x)$ (b) $\lim_{x\to 3^+} g(x)$ (c) $\lim_{x\to 3} g(x)$

15. (2.6) Find each of the limits, if possible.

(a) $\lim_{x\to -\infty} \frac{x^2 - 7x + 1}{5x - 9}$

(b) $\lim_{x\to \infty} \frac{3x^3 + 19}{2x^3 + 5x^2 - x}$

(c) $\lim_{x\to -\infty} \frac{\sqrt{9x^2 - x + 7}}{x + 26}$

16. (2.7) Use the limit definition to find the derivative.

$$\frac{d}{dx}\left[\frac{1}{x-3}\right]$$

17. (2.7) Find the equation of the tangent line to the graph of $f(x) = x^2 - 3x - 28$ at the point $x = 6$.

18. (2.7) Use the graph of f to find the value of the following:

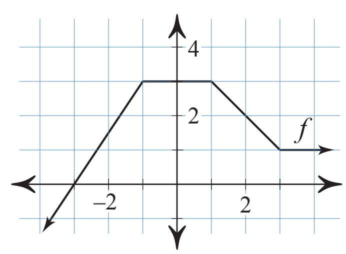
(a) $f(0)$

(b) $f'(0)$

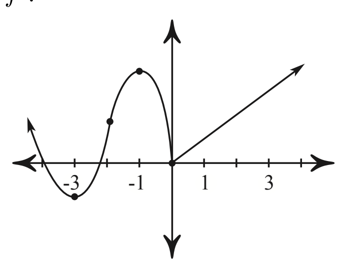
(c) $f(2)$

(d) $f'(2)$

(e) $f'(-2)$



19. The graph of f is shown below. Sketch the graph of f'.



20. (2.8) Use the definition of the derivative to prove $f'(x)$ does not exist for $f(x) = |x+1|$, at $x = -1$.

Answers

1. 2.536 feet

2. (a) $\frac{\pi}{6}, \frac{11\pi}{6}$; (b) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

3. (a) $\frac{54}{x}$; (b) $\frac{\sqrt[6]{y}}{\sqrt{x}}$

4. $f(x) = 2(\frac{1}{4})^x$

5. (a) 15; (b) $g^{-1}(x) = (x-8)^2, x \geq 8$; (c) 85

6. (a) $\ln\left(\frac{\sqrt{a}}{bc^2}\right)$; (b) $\ln\left(\frac{x^5 z^3}{y}\right)$

7. (a) -2; (b) 5; (c) 6.369

8.

x	-2.1	-2.01	-2.001
$f(x)$	-0.7881	-0.7989	-0.7999

x	-1.999	-1.99	-1.9
$f(x)$	-0.8001	-0.8012	-0.8121

$L = -\frac{4}{5}$

9. (a) -2; (b) -4; (c) d.n.e.; (d) -1; (e) 5

10. (a) $-\infty$; (b) d.n.e.; (c) ∞; (d) ∞

11. (a) $\frac{1}{2}$; (b) -6; (c) $4x$; (d) $\frac{1}{2}$

12. $L = 25$; Given any $\varepsilon > 0$, let $\delta = \frac{\varepsilon}{11}$. For $4 < x < 6$, if $|x-5| < \delta = \frac{\varepsilon}{11} \Rightarrow$
$|x^2 + 2 - 27| = |x^2 - 25| = |x+5||x-5| < 11|x-5| < (11)\left(\frac{\varepsilon}{11}\right) = \varepsilon$.

13. (a) $(-\infty, -7) \cup (-7, 7) \cup (7, \infty)$; (b) $(-\infty, \infty)$

14. (a) 3; (b) 5; (c) d.n.e.

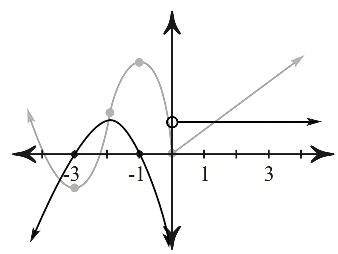
15. (a) $-\infty$; (b) $\frac{3}{2}$; (c) -3

16. $\frac{d}{dx}\left[\frac{1}{x-3}\right] = \lim_{h\to 0} \frac{\frac{1}{x+h-3} - \frac{1}{x-3}}{h} =$
$\lim_{h\to 0} \frac{x-3-(x+h-3)}{h(x+h-3)(x-3)} =$
$\lim_{h\to 0} \frac{-h}{h(x+h-3)(x-3)} =$
$\lim_{h\to 0} \frac{-1}{(x+h-3)(x-3)} = \frac{-1}{(x-3)^2}$

17. $y = 9x - 64$

18. (a) 3; (b) 0; (c) 2; (d) -1; (e) $\frac{3}{2}$

19. see graph



20. $f'(-1) = \lim_{x\to -1} \frac{|x+1| - |-1+1|}{x-(-1)} =$
$\lim_{x\to -1} \frac{|x+1|}{x+1}$
But $\lim_{x\to -1^+} \frac{|x+1|}{x+1} = 1$ and

$\lim_{x\to -1^-} \frac{|x+1|}{x+1} = -1$
Therefore, the limit does not exist as $x \to -1$, so it follows that $f'(-1)$ does not exist.