CONTENT This exam will cover the material discussed in chapter 3.

TOPICS You should be comfortable with the following topics:

Limit definition of the derivative, non-differentiability, the power rule, equation of the tangent line, horizontal tangent line, product rule, quotient rule, higher-order derivatives, chain rule, implicit differentiation, logarithmic differentiation, related rates, differentials

FORMULAS You should have the following formulas memorized.

Common Limits

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{\cos x - 1}{x}=0$$

Product Rule

$$\frac{d}{dx}[f(x)g(x)]=f'(x)g(x)+f(x)g'(x)$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$$

Chain Rule

$$\frac{d}{dx}[f(g(x))]=f'(g(x))g'(x) \quad \text{or}$$

$$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$$

The Differential

$$dy=f'(x)\,dx$$

Common Derivatives

$$\frac{d}{dx}[e^u]=e^u u' \qquad \frac{d}{dx}[\sin u]=\cos(u)u' \qquad \frac{d}{dx}[\sec u]=\sec(u)\tan(u)u'$$

$$\frac{d}{dx}[a^u]=a^u\ln(a)u' \qquad \frac{d}{dx}[\cos u]=-\sin(u)u' \qquad \frac{d}{dx}[\csc u]=-\csc(u)\cot(u)u'$$

$$\frac{d}{dx}[\ln(u)]=\frac{u'}{u} \qquad \frac{d}{dx}[\tan u]=\sec^2(u)u' \qquad \frac{d}{dx}[\arctan u]=\frac{u'}{u^2+1}$$

$$\frac{d}{dx}[\log_a u]=\frac{u'}{u\ln(a)} \qquad \frac{d}{dx}[\cot u]=-\csc^2(u)u' \qquad \frac{d}{dx}[\arcsin u]=\frac{u'}{\sqrt{1-u^2}}$$

PRACTICE PROBLEMS

1. (3.1) Find the derivative.

(a) $y=\dfrac{1}{x^2}-3\sqrt{x}+11\sqrt[3]{x^5}$

(b) $f(x)=3(x^5-e^x)$

(c) $g(x)=\dfrac{4x^4-15x^3+2}{2x^2}$

2. (3.2) Find the equation of the tangent line to the graph of the function at the indicated point.

$$f(x)=x^2-3x-28, \quad \text{at } x=6$$

3. (3.2) Find the derivative.

(a) $h(x)=e^x(x^2+4x)$ **(b)** $A(t)=\dfrac{t^2-2}{t^2+2}$

4. (3.3) Use the limit definition of the derivative to prove: $\frac{d}{dx}[\cos x]=-\sin x$.

5. (3.3) Differentiate.

(a) $h(t)=\sin^2(t)$

(b) $r(\theta)=\dfrac{\tan\theta}{\sec\theta-1}$

6. (Chap 3) Find the derivative and simplify.

(a) $g(x) = \sqrt{2x-3}$

(b) $h(t) = t(4t^2+7)^5$

(c) $f(\theta) = \sin\sqrt[3]{\theta} + \sqrt[3]{\sin\theta}$

(d) $y = x^2 e^{-x^2}$

(e) $r(x) = \ln\sqrt{x^2-81}$

7. (3.4) Find $dy/d\theta$.

$$y = \sin[\cos(\tan\theta)]$$

8. (Chap 3) Find the derivative.

(a) $f(x) = x \cdot 3^{2x}$ **(b)** $g(t) = \log_2\sqrt{x^2-1}$

9. (3.4) If $g(-1) = 2$ and $g'(-1) = 3$ find $h'(-1)$ for $h(x) = [g(x)]^5$

10. (3.5) Find dy/dx by implicit differentiation.

(a) $\sqrt{xy} = x - 2y + 1$ **(b)** $x = \ln\left(\frac{1}{y}\right)$

11. (3.5) Find the equation of the tangent line to the graph of $y^2 - x + 1 = 0$, at the point $(2, -1)$.

12. (3.5) Find y' and y'' for

$$x^4 - y^4 = 16$$

13. (3.5) Find the derivative.

(a) $f(\theta) = \arcsin\theta^2$

(b) $h(t) = \arctan(e^t)$

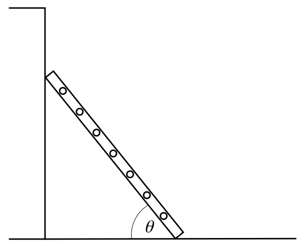
14. (3.6) Use logarithmic differentiation to find y'.

$$y = (x^2+1)^{\tan(x)}$$

15. (3.9) The length l, of a cylindrical balloon is increasing at a rate of 3 cm per second while the radius r, is increasing at 1 cm per second. Find the rate at which the volume is increasing when the length is 30 cm and the radius is 5 cm. The equation for the volume of this balloon is

$$V = \frac{4}{3}\pi r^3 + l \cdot \pi r^2$$

16. (3.9) A 6 foot ladder is sliding down a vertical wall at a rate of 1 foot per second. What is the rate of change of the angle between the ladder and the ground when the top of the ladder is 4 feet from the ground?



17. (3.10) Find Δy and dy.

$$y = \arctan x, \quad x = 1, \quad \Delta x = dx = 0.05$$

18. (3.10) Use dy with $y = \sqrt{x^2-11}$ to approximate the value of $\sqrt{5.8^2-11} = \sqrt{22.64}$ without a calculator. Express your answer as a reduced fraction.

19. (3.10) A hunter is attempting to hit a target 50 feet away. He determines the angle above horizontal should be 30° with a possible error of 1°. Use the differential to estimate the possible distance by which the hunter could miss the target. *Hint: convert dx (the error) to radians.*

Answers

1. (a) $y' = -\dfrac{2}{x^3} - \dfrac{3}{2\sqrt{x}} + \dfrac{55\sqrt[3]{x^2}}{3}$

(b) $f'(x) = 3(5x^4 - e^x)$

(c) $g'(x) = 4x - \frac{15}{2} - \frac{2}{x^3}$

2. $y = 9x - 64$

3. (a) $h'(x) = e^x(x^2 + 6x + 4)$;

(b) $A'(t) = \dfrac{8t}{(t^2+2)^2}$

4. $\frac{d}{dx}[\cos x] = \lim\limits_{h\to 0} \dfrac{\cos(x+h) - \cos(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cos(x)\cos(h) - \cos(x)}{h} - \lim\limits_{h\to 0} \dfrac{\sin(x)\sin(h)}{h}$

$= \cos(x) \lim\limits_{h\to 0} \dfrac{\cos(h) - 1}{h} - \sin x \lim\limits_{h\to 0} \dfrac{\sin(h)}{h}$

$= \cos(x)(0) - \sin x(1) = -\sin(x)$

5. (a) $h'(t) = 2\sin(t)\cos(t)$

(b) $r'(\theta) = \dfrac{\sec\theta}{1 - \sec\theta}$

6. (a) $g'(x) = \dfrac{1}{\sqrt{2x-3}}$

(b) $h'(t) = (4t^2+7)^4(44t^2+7)$

(c) $f'(\theta) = \dfrac{\cos\sqrt[3]{\theta}}{3\sqrt[3]{\theta^2}} + \dfrac{\cos\theta}{3\sqrt[3]{\sin^2\theta}}$

(d) $dy/dx = 2xe^{-x^2}(1 - x^2)$

(e) $r'(x) = \dfrac{x}{x^2 - 81}$

7. $y' = \cos[\cos(\tan\theta)](-\sin(\tan\theta))\sec^2(\theta)$

8. (a) $f'(x) = 3^{2x}(1 + 2x\ln(3))$

(b) $g'(t) = \dfrac{x}{(x^2-1)\ln(2)}$

9. 240

10. (a) $\dfrac{2\sqrt{xy} - y}{4\sqrt{xy} + x}$

(b) $-y$

11. $y = -\frac{1}{2}x$

12. $y' = \dfrac{x^3}{y^3}$; $\quad y'' = -\dfrac{48x^2}{y^7}$

13. (a) $f'(\theta) = \dfrac{2\theta}{\sqrt{1-\theta^4}}$; $\quad$ (b) $h'(t) = \dfrac{e^t}{1+e^{2t}}$

14. $(x^2+1)^{\tan(x)}\left[\sec^2(x)\ln(x^2+1) + \dfrac{2x\tan(x)}{x^2+1}\right]$

15. 475π cm^3/sec

16. θ is decreasing at a rate of $\frac{\sqrt{5}}{10}$ radians/sec

17. $\Delta y \approx 0.02439$; $\quad dy = \dfrac{1}{40}$

18. $\dfrac{119}{25}$

19. $\pm\frac{10\pi}{27} \approx \pm 1.164$ feet