CONTENT This exam will cover the material discussed in section 4.9 and chapter 5 in your book.

TOPICS You should be comfortable with the following topics:

Indefinite integral, antiderivative, solutions to a differential equation, solving a projectile in motion problem, sigma notation, upper and lower sum, right and left sum, the Riemann Sum definition of a definite integral, The Fundamental Theorem of Calculus parts 1 and 2, definite integral, integration by substitution.

FORMULAS You should have the following formulas memorized.

Common Antiderivatives

$$\int k\,du = ku + C \qquad \int a^u\,du = \frac{a^u}{\ln a} + C \qquad \int \sec^2 u\,du = \tan u + C$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \qquad \int \sin u\,du = -\cos u + C \qquad \int \csc^2 u\,du = -\cot u + C$$

$$\int \frac{1}{u}\,du = \ln|u| + C \qquad \int \cos u\,du = \sin u + C \qquad \int \sec u \tan u\,du = \sec u + C$$

$$\int e^u\,du = e^u + C \qquad \int \tan u\,du = -\ln|\cos u| + C \qquad \int \csc u \cot u\,du = -\csc u + C$$

$$\int \cot u\,du = \ln|\sin u| + C$$

$$\int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u) + C \qquad \int \frac{1}{1+u^2}\,du = \arctan(u) + C$$

Common Sums

$$\sum_{i=1}^{n} k = kn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Reimann Sum

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x, \quad \text{where } \Delta x = \frac{b-a}{n} \text{ and } x_i^* \text{ can be } x_i^* = a + i\Delta x$$

PRACTICE PROBLEMS

1. (4.9) Recover the function f if $f'(x) = 2x + 3$, and $f(2) = -1$.

2. (4.9) A car traveling 60 mph along a straight road decelerates at a constant rate of 10 ft/sec^2.

(a) How long will it take until the speed is 45 mph? [*Note:* 1 mi. = 5,280 ft.]

(b) How far will the car travel before coming to a stop?

3. (5.1) Approximate the area between $y = x - x^2$ and the x-axis, from $x = 0$ to $x = 1$, with $n = 5$, using (a) the upper sum, (b) the lower sum, (c) the left sum, and (d) the right sum. What is the actual area?

4. (5.2) Evaluate by hand.

(a) $\displaystyle\sum_{j=1}^{50} j + 2$ (b) $\displaystyle\sum_{i=1}^{n} \frac{(i-1)^2}{n^2}$

5. (5.2) Use the Riemann Sum to evaluate the definite integral.

$$\int_{-1}^{3} 2(x+1)^3 \, dx$$

6. (5.2) Use geometry to evaluate the definite integrals. Use your calculator to verify your result.

(a) $\displaystyle\int_{-1}^{1} |x| \, dx$ (b) $\displaystyle\int_{0}^{3} [\![x]\!] \, dx$

(c) $\displaystyle\int_{-2}^{2} (5 + \sqrt{4 - x^2}) \, dx$

7. (5.3) Consider the function

$$F(x) = \int_0^x \sqrt[3]{t} \, dt$$

(a) Simplify F by evaluating the integral.

(b) Find $F(1)$, $F(8)$, and $F(1000)$.

(c) Illustrate part 1 of the Fundamental Theorem of Calculus by finding $F'(x)$ using your result from part (a).

8. (5.3) Find $g'(x)$.

$$g(x) = \int_{\pi/6}^{x^4} \frac{t}{\sin t} \, dt$$

9. (5.3) Use The Fundamental Theorem of Calculus to evaluate the following definite integrals. Use your calculator to verify your result.

(a) $\displaystyle\int_{-3}^{6} (v^2 - 1) \, dv$ (b) $\displaystyle\int_{0}^{\pi} \sin\phi \, d\phi$

(c) $\displaystyle\int_{0}^{\ln 2} 0.5e^x \, dx$ (d) $\displaystyle\int_{\sqrt{3}/3}^{1} \frac{12}{1+y^2} \, dy$

10. (5.4) Evaluate the following indefinite integrals.

(a) $\displaystyle\int (4x^3 + 3\sqrt{x} - \frac{2}{x^3}) \, dx$ (b) $\displaystyle\int (\cos\theta + \csc\theta \cot\theta) \, d\theta$ (c) $\displaystyle\int \frac{7y^2 - 1}{\sqrt{y}} \, dy$

11. (5.5) Evaluate the indefinite integrals using substitution.

(a) $\displaystyle\int 4x(2 - x^2)^5 \, dx$

(b) $\displaystyle\int (\sec^2 5x)\sqrt{\tan 5x} \, dx$

(c) $\displaystyle\int \frac{1}{x-4} \, dx$ (d) $\displaystyle\int \frac{x}{\sqrt{x+2}} \, dx$

(e) $\displaystyle\int (x+1)e^{x^2+2x} \, dx$ (f) $\displaystyle\int \frac{1}{x \ln x^3} \, dx$

(g) $\displaystyle\int \tan 6\theta \, d\theta$

12. (5.5) Evaluate the following definite integrals by converting the limits of integration to u.

(a) $\displaystyle\int_{\pi/2}^{\pi} \frac{\sin\theta}{2 - \cos\theta} \, d\theta$ (b) $\displaystyle\int_{0}^{2} x^3 \sqrt[3]{x^4 + 1} \, dx$

Answers

1. $f(x) = x^2 + 3x - 11$

2. (a) 2.2 seconds

(b) 387.2 feet

3. (a) $\frac{21}{100}$; (b) $\frac{14}{125}$; (c) $\frac{4}{25}$; (d) $\frac{4}{25}$; actual: $\frac{1}{6}$

4. (a) 1375

(b) $\dfrac{2n^2 - 3n + 1}{6n}$

5. $\lim_{n\to\infty} \sum_{i=1}^{n} 2(x_i^*+1)^3 \Delta x_i = \lim_{n\to\infty} \sum_{i=1}^{n} 2(\frac{4i}{n})^3 \frac{4}{n} = \lim_{n\to\infty} \frac{512}{n^4} \sum_{i=1}^{n} i^3 = \lim_{n\to\infty} \frac{512}{n^4} \frac{n^2(n+1)^2}{4} = 128$

6. (a) 1 unit^2; (b) 3 units^2; (c) $2\pi + 20$ units^2

7. (a) $F(x) = \frac{3}{4}\sqrt[3]{x^4}$

(b) $F(1) = \frac{3}{4}; F(8) = 12; F(1000) = 7,500$

(c) $F'(x) = \sqrt[3]{x}$

8. $g'(x) = \dfrac{4x^7}{\sin(x^4)}$

9. (a) 72; (b) 2; (c) 0.5 (d) π

10. (a) $x^4 + 2\sqrt{x^3} + 1/x^2 + C$

(b) $\sin\theta - \csc\theta + C$

(c) $\frac{14}{5}\sqrt{y^5} - 2\sqrt{y} + C$

11. (a) $-\frac{1}{3}(2 - x^2)^6 + C$

(b) $\frac{2}{15}\sqrt{\tan^3(5x)} + C$

(c) $\ln|x - 4| + C$

(d) $\frac{2}{3}(x + 2)^{3/2} - 4(x + 2)^{1/2} + C$ or $\frac{2}{3}(x-4)\sqrt{x+2} + C$

(e) $\frac{1}{2}e^{x^2+2x} + C$

(f) $\frac{1}{3}\ln|\ln x| + C$

(g) $-\frac{1}{6}\ln|\cos 6\theta| + C$

12. (a) $\ln\frac{3}{2}$

(b) $\frac{3}{16}(17^{4/3} - 1)$