1. find $f'(x)$ for $f(x) = x^2 \cosh 3x$

2. Determine the area bounded by the graphs of the following two equations.

$$x = 2y - y^2, \quad x = y - 6$$

3. Write the integral for the disk method and shell method for finding the volume of the solid generated by revolving the region bounded by the graphs of the following equations about the line $x = 3$. Evaluate one of your integrals to find the volume.

$$y = x^2, \; x = 2, \; y = 0$$

4. A force of 4 pounds is needed to stretch a spring from its natural length of 8 inches to a length of 16 inches. Find the work done in stretching the spring from 1 foot in length to 2 feet.

5. Find the x-coordinate of the centroid of the region bound by the graphs of $y = ax - x^2$ and $y = 0$.

6. Find the average value of the function on the given interval.

$$f(x) = \frac{1}{\sqrt{1-x^2}}, \quad \left[\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right]$$

For exercises 7–12, evaluate each integral by hand.

7. $\displaystyle\int \frac{x^2}{x+3}\,dx$

8. $\displaystyle\int y^2 \ln y \, dy$

9. $\displaystyle\int_0^\infty x^2 e^{-2x}\,dx$

10. $\displaystyle\int \cos^3(\pi\theta)\,d\theta$

11. $\displaystyle\int \frac{x^2}{\sqrt{25-x^2}}\,dx$

12. $\displaystyle\int \frac{4}{(t-1)(t-2)(t-3)}\,dt$

13. Find the arc length of the graph of the function over the given interval.

$$f(x) = \frac{4}{5}x^{5/4}, \quad [0, 9]$$

14. Find the surface area of the solid generated by revolving the the region bounded by the graphs of the following equations about the x-axis.

$$y = 4\sqrt{x}, \; x = 5, \; x = 12, \; y = 0$$

15. Use the Midpoint Rule, Trapezoidal Rule and Simpson's Rule with $n = 4$, to approximate the value of the definite integral.

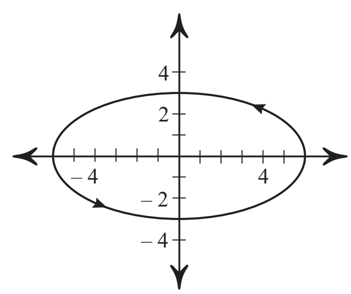
$$\int_2^3 \frac{2}{1+x^2}\,dx$$

16. Solve the differential equation.

$$\frac{dy}{dx} = y + 2$$

17. Find a set of parametric equations that represent the straight line starting at the point $(-2, 5)$ and ending at the point $(4, 1)$.

18. Find a set of parametric equations that represent the ellipse shown below.



19. Find dy/dx and d^2y/dx^2 at the point t = 2 for parametric curve:

$$x = 2 - t^3, \quad y = 4t + \ln(t)$$

20. Find two sets of polar coordinates with $0 \leq \theta < 2\pi$, that represent the same point as the rectangular coordinate, $\left(2\sqrt{3}, -2\right)$.

21. Convert the rectangular equation to polar.

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

22. Convert the polar equation to rectangular.

$$r = 6\sin(\theta)$$

23. Find all points of vertical and horizontal tangency on the polar curve.

$$r = 4 - 4\cos\theta$$

24. Find the area of the interior of the rose curve

$$r = 12\sin(3\theta)$$

25. Write an expression for the nth term of the sequence.

$$-\frac{1}{11}, \frac{3}{13}, -\frac{3}{5}, \frac{27}{17}, \ldots$$

26. Express the repeating decimal $0.0\overline{5}$ as a geometric series using sigma notation and use the sum of the series to rewrite the number as a fraction of integers.

For exercises 27–36, determine if each series converges or diverges and justify your answer. Use each test at least once.

27. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{\pi}{3}\right)^n$ **28.** $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n\ln(n)}{n}$

29. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{3n+1}{4n-5}\right)^n$ **30.** $\displaystyle\sum_{n=5}^{\infty}\frac{1}{\sqrt{n-4}}$

31. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2n-1}-\frac{1}{2n+1}$ **32.** $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}5^n}{n!}$

33. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{n^6}}$ **34.** $\displaystyle\sum_{n=1}^{\infty}ne^{-n^2}$

35. $\displaystyle\sum_{n=3}^{\infty}\frac{1}{n^3-2n^2}$ **36.** $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$

37. Find the interval and radius of convergence for the power series.

$$\sum_{n=1}^{\infty}\frac{(-1)^n(x+3)^n}{n5^n}$$

38. Find the power series for the function and determine the interval of convergence.

$$f(x) = \frac{5}{2x^2+4}$$

39. Find the Maclaurin series for the function, $f(x) = e^{x^2}$.

40. Use the Maclaurin series to approximate the value of the integral with an error of less than 0.00001.

$$\int_0^1 \sin x^3\, dx$$

1. $2x\cosh 3x + 3x^2\sinh 3x$

2. $\dfrac{125}{6}$ un^2

3. disk: $\pi\displaystyle\int_0^4 ((3-\sqrt{y})^2-1)\,dy$

shell: $2\pi\displaystyle\int_0^2 (3-x)x^2\,dx$

volume: 8π un^3

4. 60 in-lbs or 5 ft-lbs

5. $\bar{x} = \dfrac{a}{2}$

6. $\dfrac{\pi(1+\sqrt{3})}{6}$

7. $\dfrac{x^2}{2} - 3x + 9\ln(x+3) + C$

8. $\dfrac{y^3}{9}(3\ln(y)-1) + C$

9. $\dfrac{1}{4}$

10. $\dfrac{1}{3\pi}(3\sin(\pi\theta) - \sin^3(\pi\theta)) + C$

11. $\dfrac{1}{2}\left(25\arcsin\dfrac{x}{5} - x\sqrt{25-x^2}\right) + C$

12. $\ln\left[\dfrac{(t-1)^2(t-3)^2}{(t-2)^4}\right] + C$

13. $\dfrac{232}{15}$ un

14. $\dfrac{592\pi}{3}$ un^2

15. Midpoint: 2833; Trapezoidal: 0.2848; Simpson's: 0.2838

16. $y = Ce^x - 2$

17. $x = 6t-2, \quad y = -4t+5$

18. $x = 6\cos t, \quad y = 3\sin t$

19. $-\dfrac{3}{8}; \quad -\dfrac{19}{576}$

20. $\left(4, \dfrac{11\pi}{6}\right), \left(-4, \dfrac{5\pi}{6}\right)$

21. $r^2 = \dfrac{36}{9\cos^2\theta - 4\sin^2\theta}$

22. $x^2 + (y-3)^2 = 9$

23. horizontal: $(-3, 3\sqrt{3})$, $(-3, -3\sqrt{3})$
vertical: $(1, \sqrt{3})$, $(1, -\sqrt{3})$, $(-8, 0)$
cusp: $(0, 0)$

24. 36π un^2

25. $a_n = \dfrac{(-1)^n 3^{n-1}}{2n+9}$

26. $\displaystyle\sum_{n=0}^{\infty} 0.05(0.1)^n = \frac{1}{18}$

27. Diverges; geometric with $|r| = \frac{\pi}{3} > 1$

28. Converges; alternating series test

29. Converges; root test with $L = \frac{3}{4} < 1$

30. Diverges; directly compare to $\sum \frac{1}{\sqrt{n}}$

31. Converges; telescoping

32. Converges; ratio test with $L = 0 < 1$

33. Converges; p-series with $p = \frac{6}{5} > 1$

34. Converges; integral test

35. Converges; limit compare to $\sum \frac{1}{n^3}$

36. Diverges; $\lim\limits_{n\to\infty} a_n = e \neq 0$

37. $R = 5$, $(-8, 2]$

38. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 5x^{2n}}{2^{n+2}}, \quad (-\sqrt{2}, \sqrt{2})$

39. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

40. $\dfrac{449}{1920}$