Test	Series	Converges if	Diverges if	Comment
nth-term or Divergence	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	Cannot be used to show convergence
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	Sum: $S = \frac{a}{1-r}$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Integral	$\sum_{n=1}^{\infty} f(n)$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	f must be continuous, positive, and decreasing for $n \geq 1$
Direct Comparison	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq b_n$ and $\sum b_n$ converges	$0 \leq b_n \leq a_n$ and $\sum b_n$ diverges	a_n, b_n positive
Limit Comparison	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ and finite, and $\sum b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ and finite, and $\sum b_n$ diverges	a_n, b_n positive, test fails if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ or ∞
Alternating Series	$\sum_{n=1}^{\infty} (-1)^n a_n$	$\lim_{n\to\infty} a_n = 0$	$\lim_{n\to\infty} a_n \neq 0$	a_n positive and nonincreasing
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	Test fails if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$	Test fails if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$

Notes:

1. The convergent series, $\sum a_n$ converges absolutely if $\sum |a_n|$ also converges, otherwise it coverages conditionally.
2. The remainder for a convergent, alternating series is $|R_n| \leq a_{n+1}$

Determine whether each series converges or diverges and justify your answer.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{n!}$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2n+1}$

3. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n^2}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{4+|\cos n|}{n^3}$

5. $\displaystyle\sum_{n=0}^{\infty} \frac{\arctan n}{n^2+1}$

6. $\displaystyle\sum_{n=0}^{\infty} e^{(-1)^n}$

7. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2n} - \frac{1}{2n+2}\right)$

8. $\displaystyle\sum_{n=0}^{\infty} \frac{3}{4^n}$

9. $\displaystyle\sum_{n=0}^{\infty} \frac{\cos n\pi}{n+1}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0.95}}$

11. $\displaystyle\sum_{n=0}^{\infty} (\text{arcsec } n)^n$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{4/3}}{8n^2-5n+13}$

13. $\displaystyle\sum_{n=1}^{\infty} [\ln(2)]^n$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{8n-1}{\sqrt{4n^2+5n-1}}$

15. $\displaystyle\sum_{n=0}^{\infty} \frac{n}{5^n}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}}$

Answers

1. converges (ratio test)
2. diverges (limit comparison)
3. converges (alternating series test)
4. converges (direct comparison)
5. converges (integral test)
6. diverges (nth term test)
7. converges (telescoping series)
8. converges (geometric series)
9. converges (alternating series test)
10. diverges (p-series)
11. diverges (root test)
12. diverges (limit comparison)
13. converges (geometric)
14. diverges (nth term test)
15. converges (ratio test)
16. diverges (nth term)