CONTENT This exam will cover the material discussed in section 3.11 and chapter 6.

TOPICS You should be comfortable with the following topics:

Hyperbolic functions, the exponential form of hyperbolic cosine and hyperbolic sine, finding the derivative and integral of hyperbolic functions, area between curves, finding the volume of a solid of revolution using the disk method and using the shell method, work problems, average value of a function, the Mean Value Theorem for Integrals

FORMULAS You should have the following formulas memorized.

Definition of Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}$ $\cosh x = \dfrac{e^x + e^{-x}}{2}$ $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$\operatorname{csch} x = \dfrac{1}{\sinh x}$ $\operatorname{sech} x = \dfrac{1}{\cosh x}$ $\coth x = \dfrac{1}{\tanh x}$

Derivatives of Hyperbolic Functions

$\dfrac{d}{dx}[\sinh x] = \cosh x$ $\dfrac{d}{dx}[\cosh x] = \sinh x$ $\dfrac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$

$\dfrac{d}{dx}[\operatorname{csch} x] = -\operatorname{csch} x \coth x$ $\dfrac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x$ $\dfrac{d}{dx}[\coth x] = -\operatorname{csch}^2 x$

Disk Method

$V = \pi \int_a^b [R(x)]^2 \, dx$

Shell Method

$V = 2\pi \int_a^b r(x)h(x) \, dx$

Work

Work = Force × Distance

Average Value of a Function

$f_{\text{avg}} = \dfrac{1}{b-a} \int_a^b f(x) \, dx$

MVT for Integrals

$\int_a^b f(x) \, dx = f(c)(b-a)$

PRACTICE PROBLEMS

1. (3.11) Find derivative of the given function.

(a) $f(x) = \cosh^2(x^2)$ **(b)** $g(x) = \ln[\operatorname{sech} x]$

2. (Ch 5) Evaluate the integrals.

(a) $\int_0^{\ln 2} \sinh(x)\cosh(x) \, dx$

(b) $\int \dfrac{\operatorname{sech}^2(x)}{4 + \tanh(x)} \, dx$

(c) $\int x^2 e^{-x^3} \, dx$

3. (6.1) Determine the area bounded by $f(x) = 2x^3$ and $g(x) = -x^3 + x^2 + 2x$.

4. (6.2, 6.3) Use the method of your choice to find the volume of the solid formed by revolving the bounded region about the given line.

(a) $y = x^3, y = 0$ and $x = 2$, about the x-axis

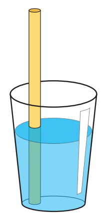
(b) $y = 2x^2 + 4x$ and $y = 0$ about the y-axis

5. (6.2, 6.3) Consider the solid formed by revolving the following bounded region about the line $x = -2$

$$y = \frac{1}{x}, y = \frac{1}{3} \text{ and } x = 1$$

Set up the integrals used for both the disk method and the shell method and evaluate one to find the volume.

6. (6.4) A 6-inch tall drinking glass that is 3 inches across the bottom and 4 inches across the top, is filled with water to a level 1-inch from the top. How much work is done when the water is sucked out through a straw if the top of the straw is 3 inches above the top of the glass? The weight of water is 0.036 lbs. per cubic inch.



7. (6.4) A 10-foot chain that weighs 4 lbs. per linear foot is coiled up on the floor with a 20 lb weight attached to the end. How much work is done if the end that is not attached to the weight is lifted to a height of 15 feet? (Assume the weight is of negligible thickness so we are lifting the weight five feet off the ground)

8. (6.5) Find the average value of the function on the given interval.

$$f(x) = \frac{\ln(x)}{x} \quad [1, 4]$$

9. (6.5) Find the value c guaranteed by the Mean Value Theorem for Integrals for the following function on the given interval.

$$f(x) = \frac{x}{\sqrt{x^2 + 16}} \quad [-3, 0]$$

Answers

1. (a) $4x\cosh(x^2)\sinh(x^2)$

(b) $-\tanh x$

2. (a) $\dfrac{9}{32}$

(b) $\ln|4+\tanh x|+C$

(c) $-\dfrac{e^{-x^3}}{3}+C$

3. $\dfrac{253}{324}$ un^2

4. (a) $\dfrac{128\pi}{7}$ un^3; (b) $\dfrac{16\pi}{3}$ un^3

5. $V = \pi\int_{1/3}^{1}\left((\frac{1}{y}+2)^2-9\right)\,dy$; $V = 2\pi\int_{1}^{3}(x+2)(\frac{1}{x}-\frac{1}{3})\,dx$; $4\pi\ln(3)-\frac{4\pi}{3}$ un^3

6. $0.036\pi\int_0^5\left(\dfrac{1}{12}y+\dfrac{3}{2}\right)^2(9-y)\,dy = \dfrac{10639\pi}{3200}\approx 10.4448$ inch-lbs

7. 500 ft-lbs.

8. $\frac{1}{6}(\ln 4)^2$

9. $c=-\sqrt{2}$