CONTENT This exam will cover the material discussed in Chapter 7.

TOPICS You should be comfortable with the following topics:

Integration by parts, the tabular method, trigonometric integrals, trigonometric substitution, partial fraction decomposition, integration tables, improper integrals.

FORMULAS You should have the following formulas memorized.

Power-Reducing Identities

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

Double Angle Identity for Sine

$\sin 2\theta = 2\sin\theta\cos\theta$

PRACTICE PROBLEMS

1. (7.1) Use integration-by-parts to evaluate.

(a) $\displaystyle\int z\sqrt{z+4}\,dz$ **(b)** $\displaystyle\int \sqrt{y}\ln y\,dy$

(c) $\displaystyle\int e^{-5x}\cos 2x\,dx$ **(d)** $\displaystyle\int \arctan(3y)\,dy$

2. (7.1) Use the tabular method of integration-by-parts to evaluate.

$$\int \theta^3 \sin(\pi\theta)\,d\theta$$

3. (7.2) Evaluate.

(a) $\displaystyle\int \sin^4 x\cos^5 x\,dx$

(b) $\displaystyle\int \tan 3y\sqrt{\sec^3 3y}\,dy$

4. (7.3) Use trigonometric substitution to evaluate the integrals.

(a) $\displaystyle\int \frac{dx}{x^2\sqrt{9-4x^2}}$

(b) $\displaystyle\int_0^4 \frac{x^3}{(16+x^2)^{3/2}}\,dx$

5. (7.4) Use partial fractions to evaluate the integrals.

(a) $\displaystyle\int \frac{2r^2+3}{r^3-2r^2+r}\,dr$

(b) $\displaystyle\int \frac{3x^2+6x-4}{(x-1)(x^2+4)}\,dx$

6. (7.3) Derive the following formula the integration tables.

$$\int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\,\operatorname{arcsec}\frac{|u|}{a} + C$$

7. (7.5) Use the formula from the previous exercise to evaluate the following integral.

$$\int \frac{\sqrt{9x^2-5}}{9x}\,dx$$

8. (7.8) Evaluate each improper integral or show that it diverges.

(a) $\displaystyle\int_0^\infty te^{-2t}\,dt$

(b) $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$

9. (7.5, 7.8) Evaluate each integral.

(a) $\displaystyle\int \frac{x}{\sqrt{x-4}}\,dx$ **(b)** $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx$

(c) $\displaystyle\int \frac{1}{\sqrt{x^2-4}}\,dx$ **(d)** $\displaystyle\int \frac{x}{x^2-4}\,dx$

(e) $\displaystyle\int \frac{1}{x^2-4}\,dx$ **(f)** $\displaystyle\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$

1. (a) $\frac{2}{15}(z+4)^{3/2}(3z-8)+C$

(b) $\frac{2}{9}y^{3/2}(3\ln y-2)+C$

(c) $\frac{1}{29}e^{-5x}(2\sin 2x-5\cos 2x)+C$

(d) $y\arctan(3y)-\frac{1}{6}\ln(1+9y^2)+C$

2. $-\dfrac{\theta^3}{\pi}\cos(\pi\theta)+\dfrac{3\theta^2}{\pi^2}\sin(\pi\theta)+\dfrac{6\theta}{\pi^3}\cos(\pi\theta)-\dfrac{6}{\pi^4}\sin(\pi\theta)+C$

3. (a) $\frac{1}{5}\sin^5 x-\frac{2}{7}\sin^7 x+\frac{1}{9}\sin^9 x+C$

(b) $\frac{2}{9}\sqrt{\sec^3 3y}+C$

4. (a) $-\dfrac{\sqrt{9-4x^2}}{9x}+C$

(b) $6\sqrt{2}-8$

5. (a) $\ln\left|\dfrac{r^3}{r-1}\right|-\dfrac{5}{r-1}+C$

(b) $\ln|x-1|+\ln(x^2+4)+4\arctan\left(\frac{x}{2}\right)+C$

6. $\displaystyle\int\frac{\sqrt{u^2-a^2}}{u}\,du=$

$$\int\frac{a\tan\theta}{a\sec\theta}a\sec\theta\tan\theta\,d\theta$$

$$=a\int\tan^2\theta\,d\theta=a\int(\sec^2\theta-1)\,d\theta$$

$$=\tan\theta-\theta+C$$

$$=\sqrt{u^2-a^2}-a\,\operatorname{arcsec}\frac{|u|}{a}+C$$

7. $\dfrac{1}{9}\left(\sqrt{9x^2-5}-\sqrt{5}\,\operatorname{arcsec}\dfrac{|3x|}{\sqrt{5}}\right)+C$

8. (a) $\dfrac{1}{4}$; (b) diverges

9. (a) (By-Parts) $2x\sqrt{x-4}-\frac{4}{3}(x-4)^{3/2}+C$;

(b) (Formula) $\arcsin\frac{x}{2}+C$;

(c) (Trig Sub) $\ln|x+\sqrt{x^2-4}|+C$;

(d) (u-Sub) $\ln\sqrt{x^2-4}+C$;

(e) (Partial Fractions) $\frac{1}{4}\ln\left|\dfrac{x-2}{x+2}\right|+C$

(f) (Improper–not correct if evaluated without using limits) 4