CONTENT This exam will cover the material discussed in Sections 8.1–8.3, 9.3, 10.1–10.4.

TOPICS You should be comfortable with the following topics:

Arc length, surface area of a solid of revolution, moments, center of mass, centroid of a planar lamina, the general and particular solutions to a differential equation, initial conditions, separable differential equations, parametric equations, converting rectangular to parametric and parametric to rectangular, graphing a parametric equation, indicating orientation, the first and second derivative of a parametric equation, arc length of a parametric equation, polar coordinates, converting polar to rectangular and rectangular to polar, graphing a polar equation, finding the slope of a polar equation, finding points of horizontal and vertical tangency, area of a polar curve, points of intersection of polar graphs.

FORMULAS You should have the following formulas memorized.

Arc Length

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Area of Surface of Revolution

$$S = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}\, dx$$

Center of Mass

$$m = \rho \int_a^b (f(x) - g(x))\, dx$$

$$M_x = \rho \int_a^b \left[\frac{f(x) + g(x)}{2}\right] (f(x) - g(x))\, dx$$

$$M_y = \rho \int_a^b x(f(x) - g(x))\, dx$$

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

Derivative of a Parametric Curve

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

2nd Derivative of a Parametric Curve

$$\frac{d^2y}{dx^2} = \frac{\dfrac{d}{dt}\left[\dfrac{dy}{dx}\right]}{\dfrac{dx}{dt}}$$

Arc Length a Parametric Curve

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Polar to Rectangular

$$x = r\cos\theta \qquad y = r\sin\theta$$

Rectangular to Polar

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$

Slope in Polar Form

$$\frac{dy}{dx} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

Area in Polar Coordinates

$$A = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2\, d\theta$$

RECOMMENDED EXERCISES

Sec 8.1 #1–20; Sec 8.2 #7–18, 30, 31, 32, 37; Sec 8.3 #25–35, 37; Ch 8 Rev #1–4, 6, 9, 10, 13–16; Sec 9.3 #1–22; Ch 9 Rev #9, 10, 11; Sec 10.1 #1–22; Sec 10.2 #1–8, 11–20, 30, 41–46, 48, 51, 52; Sec 10.3 #1–46, 54–64; Sec 10.4 #1–12, 17–42, 45–48; Ch 10 Rev #1–18, 21–26, 37–40

1. (8.1) Find the exact length of the curve,

$$y = \frac{1}{3}\sqrt{x}(x-3) \quad 1 \leq x \leq 4$$

2. (8.2) Find the area of the surface obtained by revolving the following curve about the x-axis.

$$y = \sqrt{e^x + 1} \quad 0 \leq x \leq 1$$

3. (8.3) Find m, M_x, M_y, $\bar{x}$, and $\bar{y}$ associated with calculating the center of mass of the region bounded by

$$y = 1/x, \quad y = x, \quad x = 2$$

4. (9.3) Find the general solution to differential equation. Write your answer in the form $y = f(x)$.

$$y' = y\cos x$$

5. (9.3) The rate of change of s with respect to time, t, is inversely proportional to the square root of s.

(a) Write a differential equation for the given statement.

(b) Find the general solution to this equation.

(c) If initially $s = 100$ and after six seconds $s = 144$, what would the value of s be after 10 seconds?

6. (10.1) Sketch the graph of the parametric curve.

$$x = 6\cos(t), \quad y = 3\sin(t)$$

7. (10.1) Convert the parametric equations to rectangular equations by eliminating the parameter.

(a) $x = t + 4$, $y = t^2$

(b) $x = 6\sec\theta$, $y = -1 + 3\tan\theta$

8. (10.2) Consider the parametric equation

$$x = t^2 - 4t, \quad y = \ln t$$

(a) Find the slope of the tangent line at $t = 3$

(b) Find all points of vertical tangency

(c) Find the concavity at $t = 3$

9. (10.3) Find the length of the curve represented by the parametric equations over the given interval, $0 \leq \theta \leq \pi$.

$$x = \cos\theta + \theta\sin\theta, \quad y = \sin\theta - \theta\cos\theta$$

10. (10.3) Find three sets of polar coordinates that represent the same point as the rectangular coordinate, $\left(-1, \sqrt{3}\right)$. Include at least one point with a negative radius and at least one point with a negative angle.

11. (10.3) Convert the polar equation to rectangular form.

(a) $r = 4\sec\theta$

(b) $r = 7$

(c) $r = \dfrac{2}{3\cos\theta - 4\sin\theta}$

12. (10.3) Sketch the graph of $r = 4\sin 2\theta$.

13. (10.3) Consider the polar equation, $r = \sin^2\theta$

(a) Find dy/dx

(b) Find all points of vertical and horizontal tangency.

14. (10.4) find the points of intersection of $r = 2 + \sin\theta$ and $r = 2 - \sin\theta$.

15. (10.4) Find the area outside $r = 2\sin 4\theta$ and inside $r = 2$.

1. $\dfrac{10}{3}$

2. $\pi(e+1)$

3. $m = \rho(\frac{3}{2} - \ln(2))$; $\quad M_y = \frac{4}{3}\rho$;

$M_x = \frac{11}{12}\rho$; $\quad \bar{x} = \dfrac{8}{9 - 6\ln(2)}$;

$\bar{y} = \dfrac{11}{18 - 12\ln(2)}$

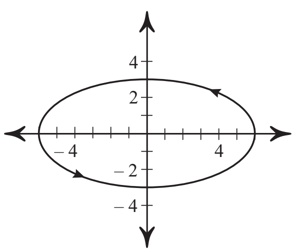
4. $y = Ce^{\sin x}$

5. (a) $\dfrac{ds}{dt} = \dfrac{k}{\sqrt{s}}$

(b) $s = \left(\dfrac{3}{2}kt + C\right)^{2/3}$

(c) $s(10) = \left(\frac{6640}{3}\right)^{2/3}$

6. see graph



7. (a) $y - x^2 + 8x - 16 = 0$

(b) $\dfrac{x^2}{36} - \dfrac{(y+1)^2}{9} = 1$

8. (a) $\frac{1}{6}$

(b) $t = 2$

(c) $-\frac{1}{9}$, concave down

9. $\dfrac{\pi^2}{2}$ units

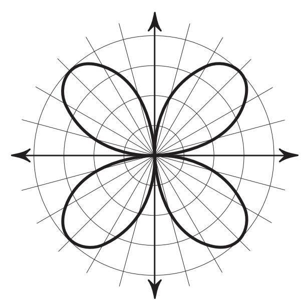
10. $(2, \frac{2\pi}{3})$, $(-2, \frac{5\pi}{3})$, $(2, -\frac{4\pi}{3})$

11. (a) $x = 4$

(b) $x^2 + y^2 = 49$

(c) $3x - 4y = 2$

12. see graph



13. (a) $\dfrac{3\sin\theta\cos\theta}{2 - 3\sin^2\theta}$

(b) horizontal: $(1, \frac{\pi}{2})$, $(1, \frac{3\pi}{2})$; vertical:$(\frac{2}{3}, \pm \arcsin \frac{\sqrt{6}}{3})$, $(\frac{2}{3}, \pi \pm \arcsin \frac{\sqrt{6}}{3})$

14. $(2, 0)$ and $(2, \pi)$

15. 2π sq. units