CONTENT This exam will cover the material discussed in Chapter 11.

TOPICS You should be comfortable with the following topics:

Finding the limit of a sequence, recursively defined sequence, finding the formula for a sequence, simplifying factorials, partial sum of series, the divergence test (or nth-term test), telescoping series, geometric series, sum of a geometric series, the integral test, p-series test, comparison test (or direct comparison test), limit comparison test, alternating series test, the remainder theorem for an alternating series, absolute and conditional convergence, ratio test, root test, power series, interval and radius of convergence, derivative and integral of a power series, Taylor and Maclaurin series.

FORMULAS You should have the following formulas memorized.

Geometric Series

$$\sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \text{ if } |r| < 1$$

Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Important Maclaurin Series

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + ..., \quad (-1, 1)$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ..., \quad (-1, 1]$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ..., \quad (-\infty, \infty)$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ..., \quad (-\infty, \infty)$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ..., \quad (-\infty, \infty)$$

PRACTICE PROBLEMS In addition to the problems below, I suggest you look at the following exercises from your textbook:

Chap 11 Rev: T/F #1–22, #1–59, omit 10, 33, 57, 58

1. (11.1) Find the limit of the sequence, if it exists.

(a) $a_n = \dfrac{2-3n}{\sqrt{n^2+1}}$

(b) $a_n = \sqrt[n]{n}$

(c) $a_n = 0, 1 - \frac{1}{2}, 1 - \frac{1}{3}, 1 - \frac{1}{4}, ...$

2. (11.1) Find a formula for the general term a_n of the sequence.

(a) $0, \frac{1}{\sqrt{2}}, \frac{2}{\sqrt{6}}, \frac{3}{2\sqrt{6}}, \frac{2}{\sqrt{30}}, \frac{5}{12\sqrt{5}} \cdots$

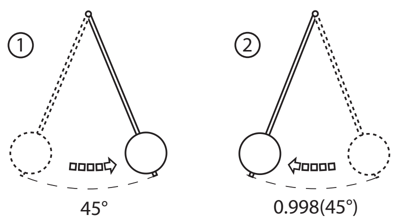
(b) $\frac{1}{2}, -\frac{3}{5}, \frac{1}{2}, -\frac{7}{17}, \frac{9}{26}, -\frac{11}{37} \cdots$

3. (11.2) Find the sum of the convergent series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n(n+4)}$

(b) $64 - 16 + 4 - 1 + ...$

4. (11.2) The first pass of a pendulum spans 45° and each subsequent pass spans 99.8% of the previous. If the pendulum continues swinging following this pattern, find the total accumulated distance traveled by a point 8 inches from the pivot point. Hint: Arc length is $S = \theta r$ where θ is in radians.



5. (11.5) Find the number of terms necessary to approximate the sum of the series with an error less than 0.0001.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \cdot 4^n} = \ln\left(\tfrac{5}{4}\right)$$

6. (11.6) Determine whether the given series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{n+1}}$ **(b)** $\displaystyle\sum_{n=1}^{\infty} \frac{\sin\left(\frac{\pi}{2}(2n-1)\right)}{n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{e^2(n+1)}$

7. (11.2 - 11.6) Show the convergence or divergence of each series. Use each of the ten methods learned in Chapter 11 at least once.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^6}$ **(b)** $\displaystyle\sum_{n=0}^{\infty} \frac{7}{0.1^n}$ **(c)** $\displaystyle\sum_{n=0}^{\infty} \frac{n}{2^n}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{3n!-1}$ **(e)** $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{e^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{4n^3-6n^2+5}$ **(g)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2-1}$

(h) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ **(i)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n-\cos^2(n)}$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n-1)}$

(k) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n}{5n+1}\right)^n$ **(l)** $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

8. (11.8) Find the radius of convergence of the power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n (x-1)^{2n}}{(2n+1)!}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n(n+1)}$

9. (11.8) Find the interval of convergence of the power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n+1}$ (b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n (x+5)^n$

10. (11.9) Find a power series for the function and find the interval of convergence.

$$f(x) = \frac{1}{2x+5}$$

11. (11.9) Use the power series for $\dfrac{1}{1+x}$, to determine a power series, centered at $c = 0$ for the function $f(x) = \ln(1+x^2)$. Identify the interval of convergence.

12. (11.10) Find the first 4 nonzero terms of the Taylor series for $f(x) = \sqrt{x}$, centered at $a = 4$.

13. (11.10) Use the Maclaurin series for $\sin(x^2)$ to approximate the value of the integral to within 0.00001. Write your answer as a reduced fraction.

$$\int_0^{1/2} \sin(x^2)\, dx$$

14. (11.10) Find the sum of the series: $\displaystyle\sum_{n=2}^{\infty} \frac{3^n}{n!}$.

1. (a) -3
(b) 1
(c) 1

2. (a) $a_n = \dfrac{n-1}{\sqrt{n!}}$
(b) $a_n = (-1)^{n+1}\dfrac{2n-1}{n^2+1}$

3. (a) $\frac{25}{12}$ (b) $\frac{256}{5}$

4. $1000\pi \approx 3{,}141$ inches

5. five terms

6. (a) absolutely convergent
(b) conditionally convergent
(c) conditionally convergent

7. *methods indicated are suggestions only. Your method may vary.*

(a) converges (p-series) (b) diverges (geometric series) (c) converges (root test) (d) diverges (nth term test) (e) diverges (ratio test) (f) converges (limit comparison) (g) converges (alternate series) (h) converges (integral test) (i) diverges (direct comparison) (j) converges (telescoping) (k) converges (root test) (l) converges (integral test)

8. (a) $R = \infty$ (b) $R = 3$

9. (a) $[-1, 1)$ (b) $\left(-\frac{19}{3}, -\frac{11}{3}\right)$

10. $f(x) = \displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{2^n x^n}{5^{n+1}}$, on $(-\frac{5}{2}, \frac{5}{2})$

11. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+2}}{n+1}$, on $[-1, 1]$

12. $f(x) \approx 2 + \dfrac{x-4}{4} - \dfrac{(x-4)^2}{64} + \dfrac{(x-4)^3}{512}$

13. $\dfrac{223}{5376}$

14. $e^3 - 4$