ODE – First-Order Applications

- 1. The number of bacteria, Q in a culture is increasing at a rate proportional to the number of bacteria present. The initial population of bacteria is $Q_0 =$ 100 and after 3 hours, Q(3) = 150.
 - (a) Find Q(t) for t > 0.
 - (b) How long will it take for the population to double? $\sum_{n=2}^{23} 2n^2$

$$\sum_{n=1} 2n$$

- 2. Suppose a substance decays at a yearly rate equal to half the square of the mass of the substance present. If the initial quantity is 50 g of the substance, how long will it be until only 25 g remain?
- 3. A person deposits \$25,000 in a bank that pays 5% per year interest, compounded continuously. The person continuously withdraws from the account at the rate of \$750 per year. Find V(t), the value of the account at time t after the initial deposit.

- 4. A fluid initially at 100°C is placed outside on a day when the temperature is 10°C, and the temperature of the fluid drops 20°C in one minute. Find the temperature T(t) of the fluid for t > 0.
- 5. An object is placed in a room where the temperature is 20°C. The temperature of the object drops by 5°C in 4 minutes and by 7°C in 8 minutes. What was the temperature of the object when it was initially placed in the room?
- 6. A container initially contains 10 L of water in which there is 20 g of salt dissolved. A solution containing 4 g/L of salt is pumped into the container at a rate of 2 L/min, and the well-stirred mixture runs out at a rate of 1 L/min. How much salt is in the tank after 40 minutes?

1. (a)
$$Q(t) = 100e^{\frac{\ln(3/2)}{3}t}$$

(b) $\frac{\ln(8)}{\ln(3/2)}$ hours

2.
$$Q(t) = \frac{50}{25t+1}; \frac{1}{25}$$
 years

3. $V(t) = 25,000 + 10,000e^{t/20}$

4.
$$T(t) = 10 + 90e^{-\ln(9/7)t}$$

$$\mathbf{5.} \ T_0 = \left(\frac{85}{3}\right)^{\circ} \mathbf{C}$$

6.
$$Q(t) = \frac{4(t+10)^2 - 200}{t+10}; Q(40) = 196 \text{ g}$$