Math 260 • Exam 1 Review

COVERAGE: The exam will cover the material discussed in lecture from Chapters 1 and 2, Sections 3.1, 4.1, 4.2, 4.5 and the first-order circuits handout.

STUDYING: Here is an overview of the topics we have covered. You should be comfortable with all of the following terms below:

Chapter 1: Differential equation, order of a DE, linear and nonlinear DE, general solution to a DE, initial conditions, initial value problem, particular solution to a DE, direction field.

Chapter 2: Linear first order equations, homogeneous linear first order equations, complementary equation, variation of parameters, integrating factors, separable equations, implicit and explicit solutions, existence and uniqueness of solutions, Bernoulli equations, homogeneous nonlinear equations, exact equations, potential function.

Section 3.1: Numerical methods, Euler's method, truncation errors, roundoff errors, Euler's semilinear method

Chapter 4: Exponential growth and decay, half-life, Newton's law of cooling, mixing problems, first-order circuits, family of curves, orthogonal trajectories.

THINGS TO BE ABLE TO DO:

- Verify that a given solution satisfies a DE and/or initial conditions.
- Solve 1st-order linear DE using variation of parameters or through the use of integrating factors
- Solve separable DE
- Know the differential equations that govern 1st-order circuits
- Solve 1st-order homogeneous DE by change of variables, using $V = \frac{y}{x}$ (i.e. $y = xV$)
- Solve Bernoulli equations
- Solve an exact DE
- Find an integrating factor in the form $\mu(x) = e^{\int p(x)}$, where $p(x) = \frac{M_y - N_x}{N}$ or $\mu(x) = e^{\int q(y)}$, where $p(y) = \frac{N_x - M_y}{M}$.
- Find an integrating factor in the form of $x^a x^b$ to make a DE exact.

Exam 1 Review Practice Problems:

1. Solve the initial value problem subject to $y(1) = e$ by using the change of variables $u = \ln y$.

$$\frac{y'}{y} - 2\frac{\ln y}{x} = \frac{1}{x}(1 - 2\ln x)$$

2. Solve the differential equation

$$x\frac{dy}{dx} - y = \frac{x^3}{y}e^{\frac{y}{x}}.$$

3. Solve the DE subject to $y(1) = 0$.

$$x\frac{dy}{dx} + y = 2x$$

4. Solve the DE

$$xy^4 dx + (y^2 + 2)e^{-3x} dy = 0.$$

5. Determine all values for the constants m and n, if there are any, for which the differential equation

$$(x^5 + y^m)dx - x^n y^3 dy = 0$$

is each of the following:

(a) Exact.

(b) Separable.

(c) Homogeneous.

(d) Linear.

(e) Bernoulli.

6. Use Euler's method to find approximate values of the solution of the initial value problem at the points $x = 1.01, 1.02$ and 1.03. Compare these to the exact values.

$$y' = xy + xy^{3/2}, \quad y(1) = 4.$$

7. A person has a fortune that grows at a rate proportional to the square root of its worth. Find the worth W of the fortune as a function of time, t if it was \$1 million six months ago and is \$4 million today.

8. Consider the RL circuit with $R = 50\,\Omega$, $L = 1\,H$, and applied voltage $E(t) = 5\,V$ and no initial current. Determine the current for $t > 0$.

9. Initially, 50 pounds of salt is dissolved in a large tank holding 300 gallons of water. A solution containing 2 pounds of salt per gallon is pumped into the tank at 3 gallons per minute and the well-stirred mixture is pumped out of the tank at 3 gallons per minute, how much salt is present after 50 minutes?

Answers

1. $y = xe^{x^2}$

2. Not homogeneous but if you let $y = xV$ it works... Solution: $e^{-y/x}(\frac{y}{x} + 1) = -x + C$

3. Divide out x and it's first order linear. Solution: $y = x - \frac{1}{x}$

4. Separable. $-\frac{1}{y} - \frac{2}{3}y^{-3} = -\frac{1}{3}xe^{3x} + \frac{1}{9}e^{3x} + C$

5. **(a)** $m = n = 0$; **(b)** $m = 0$, $n \in \mathbb{R}$; **(c)** $m = 5$, $n = 2$; **(d)** No values exist; ; **(e)** $m = 4$, $n \in \mathbb{R}$

6. Euler's method: $y_1 \approx 4.12$, $y_2 \approx 4.24608$, $y_3 \approx 4.37863$; Actual: $y = \left[1 - \frac{3}{2}e^{(1-x^2)/4}\right]^{-2} \implies$ $y_1 \approx 4.12307$, $y_2 \approx 4.25254$, $y_3 \approx 4.38887$

7. $W(t) = \$4{,}000{,}000(t + 1)^2$

8. $i(t) = \frac{1}{10}(1 - e^{-50t})$

9. $Q(50) = 600 - 550e^{-1/2}$