Math 260 • Exam 2 Review

COVERAGE: The exam will cover the material discussed in lecture from Chapter 5, Sections 6.1–6.3, the differential operator, annihilators and Cauchy-Euler equations.

STUDYING: Here is an overview of the topics we have covered. You should be comfortable with all of the following terms below:

Chapter 5: General solution to 2nd order linear differential equations, Wronskian, a fundamental set of solutions to a DE, associated homogeneous differential equation, trial solution, particular solution, method of undetermined coefficients, reduction of order technique, variation-of-parameters method.

Topics Not in the Book: Linear differential operator, annihilator method, Cauchy-Euler equations

Chapter 6: Spring problems with and without damping, 2nd-order circuits.

THINGS TO BE ABLE TO DO:

- Determine if y_1 and y_2 are linearly independent
- Be able to find the general solution to any n-th order, linear, constant coefficient, homogeneous DE by finding roots of the characteristic equation. This also means you need to understand how to handle repeated roots and complex roots that may arise.
- If initial values are given, be able to solve an initial-value problem for a specific solution.
- Know the process for finding the general solution to an 2nd-order, linear, nonhomogeneous DE. How is the solution to the nonhomogeneous DE related to the solution to the corresponding homogeneous DE?
- Be able to use annihilators to find a particular solution to a nonhomogeneous DE.
- Solve a second order DE by making the substitution $v = dy/dx$
- Be able to apply the variation-of-parameters procedure to find a particular solution to a nonhomogeneous DE.
- Be able to solve Cauchy-Euler equations.
- Be able to solve any second-order linear DE given one solution to the associated homogeneous equation using the reduction of order technique.

Exam 2 Review Practice Problems:

1. **(a)** Write the differential equation, $y'' + 4xy' - 6x^2y = x^2 \sin x$, as an operator equation and give the associated homogeneous DE.

(b) Write the DE $(D^2 + 1)(D + 3)(y) = e^{4x} + \ln(2x + 1)$ in terms of the expressions y, y', y'', etc. Is this differential equation linear? Why or why not.

2. Find a general solution to each DE

(a) $(D^2+4)^2(D+1)y = 0$

(b) $y''' + 3y'' + 3y' + y = 0$

3. Find the complementary function $y_c(x)$ and derive an appropriate trial solution for $y_p(x)$ for the given DE. Do *not* solve for the constants that arise in your trial solution.

(a) $(D+1)(D^2+1)y = 4xe^x$.

(b) $(D^2-2D+2)^3(D-2)^2(D+4)y = e^x \cos x - 3e^{2x}$.

4. Solve the DE, $y'' + 6y' + 9y = 4e^{-3x}$, by using **(a)** the method of annihilators, and **(b)** the variation of parameters method.

5. Find the general solution to the differential equation

$$y'' + 4y' + 4y = \frac{e^{-2x}}{x^2}$$

under the assumption that $x > 0$.

6. Find the general solution to the differential equation

$$x^2y'' - 3xy' - 12y = x^4 + 5x^2$$

under the assumption that $x > 0$.

7. Find the general solution to the differential equation

$$xy'' - (x+1)y' + y = 0$$

Given that $y = e^x$ is one solution.

8. Consider the RLC circuit with $R = 180\Omega$, $C = 1/280F$, $L = 20H$, and applied voltage $E(t) = 10\sin t$. Assuming no initial charge on the capacitor, but an initial current of 1 ampere. Determine the charge on the capacitor for $t > 0$.

9. A mass of one kg stretches a spring 49 cm in equilibrium. It is attached to a dashpot that supplies a damping force of 4 N for each m/sec of speed. Find the steady state component of its displacement if its subjected to an external force $F(t) = 8\sin 2t - 6\cos 2t$ N.

Solutions to Exam 2 Review Practice Problems:

1. **(a)** $(D^2+4xD-6x^2)y = x^2\sin x$, and the associated homogeneous DE is $(D^2+4xD-6x^2)y = 0$.

(b) $y''' + 3y'' + y' + 3y = e^{4x} + \ln(2x+1)$, and this differential equation **IS LINEAR**.

2. **(a)** $y(x) = c_1e^{-x} + c_2\cos 2x + c_3\sin 2x + c_4x\cos 2x + c_5x\sin 2x$

(b) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3x^2e^{-x}$

3. **(a)** $y_c(x) = c_1e^{-x} + c_2\cos x + c_3\sin x$; $y_p(x) = Ae^x + Bxe^x$

(b) $y_c(x) = c_1e^{-4x} + c_2e^{2x} + c_3xe^{2x} + c_4e^x\cos x + c_5e^x\sin x + c_6xe^x\cos x + c_7xe^x\sin x + c_8x^2e^x\cos x + c_9x^2e^x\sin x$; $y_p(x) = Ax^3e^x\cos x + Bx^3e^x\sin x + Cx^2e^{2x}$

4. $y(x) = c_1e^{-3x} + c_2xe^{-3x} + 2x^2e^{-3x}$

5. $y(x) = e^{-2x}(c_1 + c_2x - \ln x)$

6. $y(x) = c_1x^6 + c_2x^{-2} - \frac{1}{12}x^4 - \frac{5}{16}x^2$

7. $y(x) = c_1e^x + c_2(x+1)$

8. $q'' + 9q' + 14q = \frac{1}{2}\sin t, \quad q(0) = 0, q'(0) = 1 \implies q(t) = \frac{1}{500}(110e^{-2t} - 101e^{-7t} + 13\sin t - 9\cos t)$

9. $y'' + 4y' + 20y = 8\sin 2t - 6\cos 2t \implies y_p = A\cos 2t + B\sin 2t$. Solve for A and B to get: $y_p = \frac{1}{4}(\sin 2t - 2\cos 2t)$ meters.