## Math 260 • Exam 3 Review

**COVERAGE:** The exam will cover the material discussed in lecture from chapters 7 and 8.

**STUDYING:** Here is an overview of the topics we have covered. You should be comfortable with all of the following terms below:

**Chapter 7:** Power series, recursive sequence, ordinary point, singular point, regular singular point, Method of Frobenius

**Chapter 8:** Laplace transform, piecewise continuous function, exponential order, inverse Laplace transform, first shifting theorem, unit step function, second shifting theorem, convolution product, convolution integral, convolution theorem, impulse functions, Dirac $\delta$ function

### THINGS TO BE ABLE TO DO:

- Determine two linearly independent power series solutions to a differential equation
- Determine the form of the solution using the the Method of Frobenius based on the roots of the indicial equation.
- Determine the Laplace transform of a given function from the definition and from a table of common Laplace transforms
- Determine the inverse Laplace transform of a given function
- Determine the Laplace transform of a given periodic function
- Use Laplace transforms to solve initial-value problems
- Apply the first shifting theorem to compute the Laplace transform of $e^{at}f(t)$
- Use the first shifting theorem to compute the inverse Laplace transform of $F(s-a)$
- Sketch functions that involve the unit step function
- Express appropriate functions in terms of unit step functions
- Apply the second shifting theorem to compute inverse Laplace transforms that result in unit step functions
- Solve initial-value problems that involve unit step functions
- Compute $f * g$
- Use the convolution theorem to compute the Laplace transform of a convolution product
- Use the convolution theorem to compute the inverse Laplace transform of a product of functions
- Solve initial value problems involving the Dirac $\delta$ function.

### Exam 3 Review Practice Problems:

**1.** Using the power series method, find the general solution to the differential equation.

$$y'' + xy' + 3y = 0$$

**2.** Using the Method of Frobenius, find $y_1$ and the form of $y_2$. Solve for $a_n$ but not $b_n$.

$$x^2y'' + x(3+x)y' + (1+3x)y = 0$$

**3.** Use the definition to find the Laplace transform of $f(t) = t\sin 3t$.

**4.** Find $\mathcal{L}[f]$.

**(a)** $f(t) = 5\cos 2t - 7e^{-t} - 3t^6$

**(b)** $f(t) = e^{3t}\cos 5t - e^{-t}\sin 2t$

**(c)** $f(t) = 2(t-5)u(t-5)$

**5.** Find $\mathcal{L}^{-1}[F]$.

**(a)** $F(s) = \dfrac{4s+5}{s^2-9}$

**(b)** $F(s) = \dfrac{s-2}{s^2+2s+2}$

**(c)** $F(s) = \dfrac{2}{s(s^2+16)}$

**6.** Use the Laplace transform to solve the given initial-value problem.

$$y'' + 9y = 8\cos 3t, \quad y(0) = 1, \quad y'(0) = 0$$

**7.** Express $f$ using the unit step function, and find $\mathcal{L}[f]$.

$$f(t) = \begin{cases} 2 & 0 \le t < 1 \\ 1-t & 1 \le t < 2 \\ 0 & t \ge 2 \end{cases}$$

**8.** Use the Laplace transform to solve the given initial-value problem.

$$y' - 3y = f(t), \quad y(0) = 2, \text{ where}$$

$$f(t) = \begin{cases} \sin t & 0 \le t < \pi/2 \\ 1 & t \ge \pi/2 \end{cases}$$

**9.** Determine $\mathcal{L}^{-1}[F(s)G(s)]$ (a) using the convolution theorem, and (b) using partial fractions.

$$F(s) = \frac{1}{s}, \quad G(s) = \frac{1}{s-2}$$

**10.** Use the Laplace transform to solve the given initial-value problem.

$$y'' + 2y' + y = \delta(t-4), \quad y(0) = 0, \quad y'(0) = 0$$

**Answers to Exam 3 Review Practice Problems:**

**1.** $y_1 = \sum_{n=0}^{\infty} (-1)^n \frac{1 \cdot 3 \cdots (2n+1)}{(2n)!} x^{2n}$; $y_2 = \sum_{n=0}^{\infty} (-1)^n \frac{2^n (n+1)!}{(2n+1)!} x^{2n+1}$

**2.** $y_1 = x^{-1} \left[ 1 + \sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n!} x^n \right]$; $y_2 = y_1 \ln x + \sum_{n=1}^{\infty} b_n x^{n-1}$

**3.** $\frac{6s}{(s^2+9)^2}$

**4.** **(a)** $F(s) = \frac{5s}{s^2+4} - \frac{7}{s+1} - \frac{2160}{s^7}$

**(b)** $F(s) = \frac{s-3}{(s-3)^2+25} - \frac{2}{(s+1)^2+4}$

**(c)** $F(s) = \frac{2e^{-5s}}{s^2}$

**5.** **(a)** $f(t) = 4\cosh(3t) + \frac{5}{3}\sinh(3t)$

**(b)** $f(t) = e^{-t}\cos t - 3e^{-t}\sin t$

**(c)** $f(t) = \frac{1}{8}(1 - \cos 4t)$

**6.** $y(t) = \cos 3t + \frac{4}{3} t \sin 3t$

**7.** $f(t) = 2 - (1+t)u(t-1) - (1-t)u(t-2)$; $\frac{2}{s} - \frac{2s+1}{s^2}e^{-s} + \frac{s+1}{s^2}e^{-2s}$

**8.** $y(t) = \frac{1}{30}\left[63e^{3t} - 3\cos t - 9\sin t + u(t-\pi/2)(e^{3(t-\pi/2)} - 10 + 9\sin t + 3\cos t)\right]$

**9.** $\frac{1}{2}(e^{2t} - 1)$

**10.** $y(t) = u(t-4)\left[(t-4)e^{-(t-4)}\right]$