Chapter 1

Line and Angle Relationships

1.1   Sets, Statements and Reasoning

DEFINITIONS

1. A **statement** is a set of words and/or symbols that collectively make a claim that can be classified as true or false.

2. The **negation** of a statement P is the claim opposite of the original statement, indicated by "not P" or $\sim P$.

3. A **compound statement** is formed by combining multiple statements.

4. In a **conditional statement**, such as "If P, then Q" or $P \Rightarrow Q$, P is the **hypothesis** and Q is the **conclusion**.

5. **Reasoning** is a process that leads to a conclusion.

6. **Intuition** is the type of reasoning in which inspiration leads to a statement of theory.

7. **Induction** is the type of reasoning in which the use of specific observations or experiments leads to a general conclusion.

8. **Deduction** is the type of reasoning in which the knowledge and acceptance of selected assumptions guarantee the truth of a particular conclusion.

1.3   Early Definitions and Postulates

DEFINITIONS

1. An **isosceles triangle** is a triangle that has two congruent sides.

2. A **line segment** is the part of a line that consists of two points, known as *endpoints* and all points between them.

3. The **distance** between two points A and B is the length of the line segment $\overline{AB}$ that joins the two points.

4. **Congruent segments** ($\cong$) are two segments that have the same length.

5. The **midpoint** of a line segment is the point that separates the line segment into two congruent parts.

6. **Ray** AB ($\overrightarrow{AB}$), is the union of $\overline{AB}$ and all points X on $\overleftrightarrow{AB}$ such that B is between A and X.

7. **Parallel lines** are lines that lie in the same plane but do not intersect.

Postulates

1. Through two distinct points, there is exactly one line.

2. (**Ruler Postulate**) The measure of any line segment is a unique positive number.

3. (**Segment-Addition Postulate**) If X is a point of $\overline{AB}$ and A-X-B, then $AX + XB = AB$.

4. If two lines intersect, they intersect at a point.

5. Through three noncollinear points, there is exactly one plane.

6. If two distinct planes intersect, there intersection is a line.

7. Given two distinct points in a plane, the line containing these points also lies in the plane.

THEOREMS

1.3.1 The midpoint of a line segment is unique.

CONSTRUCTIONS

1. Construct a segment congruent to a given segment.

2. Construct the midpoint M of a given line segment $\overline{AB}$

1.4 Angles and Their Relationships

DEFINITIONS

8. An **angle** is the union of two rays that share a common endpoint.

9. **Congruent angles** ($\cong \angle$s) are two angles with the same measure.

10. The **bisector** of an angle is the ray that separates the given angle into two congruent angles.

11. Two angles are **complementary** if the sum of their measures is 90°. Each angle in the pair is known as the **complement** of the other.

12. Two angles are **supplementary** if the sum of their measures is 180°. Each angle in the pair is known as the **supplement** of the other.

Postulates

8. **(Protractor Postulate)** The measure of an angle is a unique positive number.

9. **(Angle-Addition Postulate)** If point D lies in the interior of $\angle ABC$, then $\text{m}\angle ABD + \text{m}\angle DBC = \text{m}\angle ABC$.

Theorems

1.4.1 There is one and only one angle bisector for a given angle.

Constructions

3. Construct an angle congruent to a given angle.

4. Construct the bisector of a given angle.

1.6   Relationships: Perpendicular Lines

DEFINITIONS

13. **Perpendicular lines** are two lines that meet to form congruent adjacent angles.

THEOREMS

1.6.1 If two lines are perpendicular, then they meet to form right angles.

1.6.2 If two lines intersect, then the vertical angles formed are congruent.

1.6.3 In a plane, there is exactly one line perpendicular to a given line at any given point.

1.6.4 The perpendicular bisector of a line segment is unique.

Constructions

5. Construct the perpendicular bisector of a given line.

1.7   The Formal Proof of a Theorem

THEOREMS

1.7.1 If two lines meet to form a right angle, then these lines are perpendicular.

1.7.2 If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.

1.7.3 If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.

1.7.4 Any two right angles are congruent.

1.7.5 If the exterior sides of two adjacent acute angles form perpendicular rays, then these angles are complementary.

1.7.6 If the exterior sides of two adjacent angles form a straight line, then these angles are supplementary.

1.7.7 If two line segments are congruent, then their midpoints separate these segments into four congruent segments.

1.7.8 If two angles are congruent, then their bisectors separate these angles into four congruent angles.