# Chapter 5

# Triangles

## 5.3 Proving Triangles Similar

### Definitions

30. (From 5.2) Two polygons are **similar** if and only if the following two conditions are satisfied:

    **1.** All pairs of corresponding angles are congruent.

    **2.** All pairs of corresponding sides are proportional.

### Postulates

15. If three angles of one triangle are congruent to three angles of another triangle, then the triangles are similar (AAA).

## Theorems, Corollaries and Lemmas

5.3.1 If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar (AA).

5.3.2 The lengths of the corresponding altitudes of similar triangles have the same ratio as the lengths of any pair of corresponding sides.

5.3.3 (SAS$\sim$) If an angle of one triangle is congruent to an angle of second triangle and the pairs of sides including the angles are proportional, then the triangles are similar.

5.3.4 (SSS$\sim$) If the three sides of one triangle are proportional to the three sides of a second triangle, then the triangles are similar.

5.3.5 If a line segment divides two sides of a triangle proportionally, then this line segment is parallel to the third side of the triangle.

## 5.4 The Pythagorean Theorem

### Definitions

31. A **Pythagorean triple** is a set of three natural numbers $(a, b, c)$ for which $a^2+b^2 = c^2$.

### Theorems, Corollaries and Lemmas

5.4.1 The altitude drawn to the hypotenuse of a right triangle separates the right triangle into two right triangles that are similar to each other and to the original triangle.

5.4.2 The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

5.4.3 The length of each leg of a right triangle is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to that leg.

5.4.4 (Pythagorean Theorem) The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.

5.4.5 (Converse of the Pythagorean Theorem) If $a$, $b$, and $c$ are the lengths of the sides of a triangle, with $c$ the length of the longest side, and if $c^2 = a^2 + b^2$, then the triangle is a right triangle with the right angle opposite the side of length $c$.

5.4.6 If the hypotenuse and leg of one right triangle are congruent to the hypotenuse and leg of a second right triangle, then the triangles are congruent (HL).

5.4.7 Let $a$, $b$, and $c$ represent the lengths of the three sides of a triangle, with $c$ the length of the longest side.

(a) If $c^2 > a^2 + b^2$, the the triangle is obtuse and the obtuse angle lies opposite the side of length $c$.

(b) If $c^2 < a^2 + b^2$, the the triangle is acute.

## 5.5 Special Right Triangles

### Theorems, Corollaries and Lemmas

5.5.1 (45-45-90 Theorem) In a triangle whose angles measure 45°, 45°, and 90°, the hypotenuse has a length equal to the product of $\sqrt{2}$ and the length of either leg.

5.5.2 (30-60-90 Theorem) In a triangle whose angles measure 30°, 60°, and 90°, the hypotenuse has a length equal to twice the length of the short leg, and the length of the longer leg is the product of $\sqrt{3}$ and the length of the shorter leg.

5.5.3 (Converse of 45-45-90 Theorem) If the length of the hypotenuse of a right triangle equals the product of $\sqrt{2}$ and the length of the either leg, then the angles of the triangle measure 45°, 45°, and 90°.

5.5.4 (Converse of 30-60-90 Theorem) If the length of the hypotenuse of a right triangle is twice the length of one leg of the triangle, then the angle of the triangle opposite that leg measures 30°.

## 5.6 Segments Divided Proportionally

### Theorems, Corollaries and Lemmas

5.6.1 If a line is parallel to one side of a triangle and intersects the other two sides, then it divides these sides proportionally.

5.6.2 When three or more parallel lines are cut by a pair of transversals, the transversals are divided proportionally by the parallel lines.

5.6.3 (The Angle-Bisector Theorem) If a ray bisects one angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the two sides that form the bisected angle.

5.6.4 (Ceva's Theorem (*pronounced "Chay'va"*)) Let point D be any point in the interior of $\triangle$ABC, and let $\overline{\text{AE}}$, $\overline{\text{BF}}$, and $\overline{\text{CG}}$ be the line segments determined by D and vertices of $\triangle$ABC. Then the product of the ratios of the lengths of the segments of each of the three sides (taken in order from a given vertex of the triangle) equals 1. In formula form:

$$\frac{\text{AG}}{\text{GB}} \cdot \frac{\text{BE}}{\text{EC}} \cdot \frac{\text{CF}}{\text{FA}} = 1$$