

# Chapter 6

## Circles

### 6.1 Circles and Related Segments and Angles

#### DEFINITIONS

32. A **circle** is the set of all points in a plane that are a fixed distance from a given point known as the center of the circle.
  
  
  
  
  
  
  
  
  
  
33. **Congruent circles** are two or more circles that have congruent radii.
  
  
  
  
  
  
  
  
  
  
34. **Concentric circles** are coplanar circles that have a common center.

35. A **central angle** of a circle is an angle whose vertex is the center of the circle and whose sides are radii of the circle.
36. In a circle or congruent circles, **congruent arcs** are arcs with equal measures.
37. An **inscribed angle** of a circle is an angle whose vertex is a point on the circle and whose sides are chords of the circle.

## POSTULATES

16. (Central Angle Postulate) In a circle, the degree measure of a central angle is equal to the degree measure of its intercepted arc.
17. (Arc-Addition Postulate) If B lies between A and C on a circle, then  $m\widehat{AB} + m\widehat{BC} = m\widehat{ABC}$

## THEOREMS, COROLLARIES AND LEMMAS

6.1.1 A radius that is perpendicular to a chord bisects the chord.

6.1.2 The measure of an inscribed angle of a circle is one-half the measure of its intercepted arc.

6.1.3 In a circle (or congruent circles), congruent minor arcs have congruent central angles.

6.1.4 In a circle (or congruent circles), congruent central angles have congruent arcs.

6.1.5 In a circle (or congruent circles), congruent chords have congruent minor chords.

6.1.6 In a circle (or congruent circles), congruent arcs have congruent chords.

6.1.7 Chords that are the same distance from the center of a circle are congruent.

6.1.8 Congruent chords are located at the same distance from the center of the circle.

6.1.9 An angle inscribed in a semicircle is a right angle.

6.1.10 If two inscribed angles intercept the same arc, then these angles are congruent.

## 6.2 More Angle Measures in the Circle

### DEFINITIONS

38. A **tangent** is a line that intersects a circle at exactly one point; the point of intersection is the **point of contact** or **point of tangency**.
39. A **secant** is a line (or segment or ray) that intersects a circle at exactly two points.
40. A polygon is **inscribed in a circle** if its vertices are points on the circle and its sides are chords of the circle. Equivalently, the circle is said to be **circumscribed about the polygon**. The polygon inscribed in a circle is further described as a **cyclic polygon**.
41. A polygon is **circumscribed about a circle** if all sides of the polygon are line segments tangent to the circle; also, the circle is said to be **inscribed in the polygon**.

## THEOREMS, COROLLARIES AND LEMMAS

6.2.1 If a quadrilateral is inscribed in a circle, the opposite angles are supplementary.

*Alternative Form:* The opposite angles of a cyclic quadrilateral are supplementary

6.2.2 The measure of an angle formed by two chords that intersect within a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

6.2.3 The radius (or any other line through the center of a circle) drawn to a tangent at the point of tangency is perpendicular to the tangent at that point.

6.2.4 The measure of an angle formed by a tangent and a chord drawn to the point of tangency is one-half the measure of the intercepted arc.

6.2.5 The measure of an angle formed when two secants intersect at a point outside the circle is one-half the difference of the measures of the two intercepted arcs.

6.2.6 If an angle is formed by a secant and a tangent that intersects in the exterior of a circle, then the measure of the angle is one-half the difference of the measures of its intercepted arcs.

6.2.7 If an angle is formed by two intersecting tangents, then the measure of the angle is one half the difference of the measures of the intercepted arcs.

6.2.8 If two parallel lines intersect a circle, the intercepted arcs between these lines are congruent.

## 6.3 Line and Segment Relationships in the Circle

### DEFINITIONS

42. For two circles with different centers, the **line of centers** is the line (or line segment) containing the centers of both circles.

### THEOREMS, COROLLARIES AND LEMMAS

- 6.3.1 If a line is drawn through the center of a circle perpendicular to a chord, then it bisects the chord.

- 6.3.2 If a line through the center of a circle bisects a chord other than a diameter, then it is perpendicular to the chord.

- 6.3.3 The perpendicular bisector of a chord contains the center of the circle.



6.3.4 The tangent segments to a circle from an external point are congruent.

6.3.5 If two chords intersect within a circle, then the product of the lengths of the segments (parts) of one chord is equal to the product of the lengths of the segments of the other chord.

6.3.6 If two secant segments are drawn to a circle from an external point, then the product of the lengths of each secant with its external segment are equal.

6.3.7 If a tangent segment and a secant segment are drawn to a circle from an external point, then the square of the length of the tangent equals the product of the length of the secant with the length of its external segment.