Chapter 8

Areas of Polygons and Circles

8.1 Area and Initial Postulates

Postulates

18. Corresponding to every bounded region is a unique positive number A, known as the area of the region.

19. If two closed figures are congruent, then their areas are equal.

20. Let R and S be two enclosed region that do not overlap. Then

$$A_{R \cup S} = A_R + A_S$$

21. The area A of a rectangle whose base has length b and whose altitude has length h is given by

$$A = bh.$$

Theorems, Corollaries and Lemmas

8.1.1 The area A of a square whose sides are each of length s is given by

$$A = s^2$$

8.1.2 The area A of a parallelogram with a base of length b and with corresponding altitude of length h is given by

$$A = bh.$$

8.1.3 The area A of a triangle whose base has length b and whose corresponding altitude has length h is given by

$$A = \frac{1}{2}bh.$$

8.1.4 The area of a right triangle with legs of lengths a and b is given by

$$A = \frac{1}{2}ab.$$

8.2 Perimeter and Area of Polygons

Definitions

50. The **perimeter** of a polygon is the sum of the lengths of all sides of the polygon.

Theorems, Corollaries and Lemmas

8.2.1 (Heron's Formula) If three sides of a triangle have lengths a, b, and c, then the area A of the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s \text{ is the semiperimeter, } s = \tfrac{1}{2}(a+b+c).$$

8.2.2 (Brahmagupta's Formula) for a cyclic quadrilateral with sides of lengths a, b, c and d, the area is given by

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}, \text{ where } s \text{ is the semiperimeter, } s = \tfrac{1}{2}(a+b+c+d).$$

8.2.3 The area A of a trapezoid whose bases have lengths b_1 and b_2 and whose altitude has length h is given by

$$A = \tfrac{1}{2}(b_1 + b_2)h.$$

8.2.4 The area of any quadrilateral with perpendicular diagonals of lengths d_1 and d_2 is given by

$$A = \tfrac{1}{2} d_1 d_2.$$

8.2.5 The area A of a rhombus whose diagonals have lengths d_1 and d_2 is given by

$$A = \tfrac{1}{2} d_1 d_2.$$

8.2.6 The area A of a kite whose diagonals have lengths d_1 and d_2 is given by

$$A = \tfrac{1}{2} d_1 d_2.$$

8.2.7 The ratio of the areas of two similar triangles equals the square of the ratio of the lengths of any two corresponding sides.

$$\frac{A_1}{A_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{b_1}{b_2}\right)^2 = \left(\frac{c_1}{c_2}\right)^2$$

8.4 Circumference and Area of a Circle

Postulates

22. The ratio of the circumference to a circle to the length of its diameter is a unique positive constant.

Definitions

51. $\boldsymbol{\pi}$ is the ratio between the circumference C and the diameter length d of any circle; thus $\pi = \frac{C}{d}$ in any circle.

Theorems, Corollaries and Lemmas

8.4.1 The circumference of a circle is given by

$$C = \pi d \quad \text{or} \quad C = 2\pi r.$$

8.4.2 In a circle whose circumference is C, the length ℓ of an arc whose degree measure is m is given by

$$\ell = \frac{m}{360^\circ} \cdot C \quad \text{or} \quad \ell\widehat{\mathrm{AB}} = \frac{\mathrm{m}\widehat{\mathrm{AB}}}{360^\circ} \cdot C$$

8.4.3 The area A of a circle whose radius has length r is given by

$$A = \pi r^2.$$

8.5 More Area Relationships in the Circle

Postulates

23. The ratio of the degree measure m of an arc (or central angle) of a sector to 360° is the same as the ratio of the area of the sector to the area of the circle; that is

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{m}{360^\circ}$$

Definitions

52. A **sector** of a circle is a region bounded by two radii of the circle and an arc intercepted by those radii.

53. A **segment** of a circle is a region bounded by a chord and its minor (or major) arc.

Theorems, Corollaries and Lemmas

8.5.1 In a circle of radius r, the area A of a sector whose arc has degree measure m, is given by

$$A = \frac{m}{360^\circ}\pi r^2$$

8.5.2 The area of a semicircular region of radius r is $A = \frac{1}{2}\pi r^2$.

8.5.3 Where P represents the perimeter of a triangle and r represents the length of the radius of its inscribed circle, the area of the triangle is given by

$$A = \frac{1}{2}rP$$