Chapter 9

Surfaces and Solids

9.1 Prisms, Area, and Volume

Postulates

24. (Volume Postulate) Corresponding to every solid is a unique positive number V known as the volume of that solid.

25. The volume of a right rectangular prism is given by $V = \ell wh$, where ℓ measures the length, w the width, and h the altitude of the prism.

26. The volume of a right prism is given by $V = Bh$, where B is the area of the base and h is the altitude of the prism.

Definitions

54. A **right prism** is a prism in which the lateral edges are perpendicular to the base edges at their points of intersection.

55. An **oblique prism** is a prism in which the parallel lateral edges are oblique to the base edges at their points of intersection.

56. The **lateral area** L of a prism is the sum of the areas of all lateral faces.

57. For any prism, the **total area** T is the sum of the lateral area and the areas of the bases.

58. A **regular prism** is a right prism whose bases are regular polygons.

59. A **cube** is a right square prism whose edges are congruent.

Theorems, Corollaries and Lemmas

9.1.1 The lateral area L of any prism whose altitude has measure h and whose base has perimeter P is given by $L = hP$.

9.1.2 The total area T of any prism with lateral area L and base area B is given by $T = L + 2B$.

9.2 Pyramids, Area, and Volume

Definitions

60. A **regular pyramid** is a pyramid whose base is a regular polygon and whose lateral edges are all congruent.

61. The **slant height** of a regular pyramid is the altitude from the vertex of the pyramid to the base of any of the congruent lateral faces of the regular pyramid.

Theorems, Corollaries and Lemmas

9.2.1 In a regular pyramid, the length a of the apothem of the base, the altitude h, and the slant height ℓ satisfy the Pythagorean Theorem; that is $\ell^2 = a^2 + h^2$, in every regular pyramid.

9.2.2 The lateral area L of a regular pyramid with slant height of length ℓ and perimeter P of the base is given by $L = \frac{1}{2}\ell P$

9.2.3 The total area (surface area) T of a pyramid with lateral area L and base area B is given by $T = L + B$.

9.2.4 The volume V of a pyramid having base area B and an altitude of length h is given by $V = \frac{1}{3}Bh$.

9.2.5 In a regular pyramid, the lengths of the altitude h, radius r of the base and lateral edge e, satisfy the Pythagorean Theorem; that is $e^2 = h^2 + r^2$.

9.3 Cylinders and Cones

Theorems, Corollaries and Lemmas

9.3.1 The lateral area L of a right circular cylinder with altitude of length h and circumference C of the base is given by $L = hC$ (*alternate form:* $L = 2\pi rh$).

9.3.2 The total area T of a right circular cylinder with base area B and lateral area L is given by $T = L + 2B$ (*alternate form:* $T = 2\pi rh + 2\pi r^2$).

9.3.3 The volume V of a right circular cylinder with base area B and altitude of length h is given by $V = Bh$ (*alternate form:* $V = \pi r^2 h$).

9.3.4 The lateral area L of a right circular cone with slant height of length ℓ and circumference C of the base is given by $L = \frac{1}{2}\ell C$ (*alternate form:* $L = \pi r \ell$).

9.3.5 The total area T of a right circular cone with base area B and lateral area L is given by $T = B + L$ (*alternate form:* $T = \pi r^2 + \pi r \ell$).

9.3.6 In a right circular cone, the lengths of the radius r (of the base), the altitude h, and the slant height ℓ satisfy the Pythagorean Theorem; that is, $\ell^2 = r^2 + h^2$ in every right circular cone.

9.3.7 The volume of a right circular cone with base area B and altitude of length h is given by $V = \frac{1}{3}Bh$ (*alternate form:* $V = \frac{1}{3}\pi r^2 h$).

9.4 Polyhedrons and Spheres

Definitions

62. A **regular polyhedron** is a convex polyhedron whose faces are congruent regular polygons arranged in such a way that adjacent faces form congruent dihedral angles.

Theorems, Corollaries and Lemmas

9.4.1 (Euler's equation) The number of vertices V, the number of edges E, and the number of faces F of a polyhedron are related by the equation

$$V + F = E + 2$$

9.4.2 The surface area S of a sphere whose radius is r is given by $S = 4\pi r^2$.

9.4.3 The volume V of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.