(a) Go to desmos.com/calculator, and enter a function called $y$:

$$y = at^3 - bt^2$$

The site will ask if you’d like sliders for the constants $a$ and $b$: click “yes.”

(b) Adjust the sliders for $a$ and $b$ so that both are positive, and so that the red graph crosses the horizontal axis at $t = 6$. Analytically, what is the relationship between $a$ and $b$ so that this happens? Are the values that you adjusted the slider to consistent with this condition? You’ll keep these values of $a$ and $b$ for the rest of this exercise.

(c) Now, enter a new function $v$. For $v$, enter the derivative of $y$ with respect to $t$. Look at the two graphs, and individually write down three things you notice about the relationship between the red graph ($y$) and the derivative graph ($v$). Share your properties with the group and see if everyone agrees.

(d) By counting squares, what’s the area under the velocity graph from $t = 0$ to $t = 4$? Compare to $\int_0^4 v(t)dt = y(4) - y(0)$, and make sure the two are close.

(e) Now, the physics: the red graph is describing where a dog is along a line. Create a story for what the dog is doing from $t = 0$ to $t = 7$.

(f) The blue graph is the velocity of the dog as a function of time (hence us calling it $v$). Is this consistent with your story from the previous part?

Now, instead of taking a graph and making up a story to go with it, now we’re going to take a story and make graphs from it (really, two stories of two blocks from the following picture):

Imagine that the above picture shows the location of two blocks (“a” on top and “b” on bottom) at times $t = 1$, $t = 2$, ..., $t = 7$. The tickmarks go from $x = 0$ on the left end to $x = 30$ on the right (so that, for example, both blocks are somewhere between $x = 15$ and $x = 16$ at $t = 5$).

(a) First of all, from the picture alone try to determine when (at what $t$ value(s)) do the two blocks have the same speed.

(b) $x_a = (1/2)t^2 + (1/2)t + (3/4)$ is a quadratic function of $t$ describing block a. Find $x_b$ as a function of time (should be linear, since the points are evenly spread out).

(c) Find the derivatives of the two functions – call these $v_a$ and $v_b$. Plot the two in Desmos and see when they cross. Is this $t$ value the same as what you said in part (a)? Explain.