1. A store sells rope by the whole foot. If a landscaper needs a rope at least 16,800 mm long, what is the least number of feet she must purchase? Use 1 inch = 2.54 cm.
A. 6 feet B. 55 feet C. 56 feet D. 552 feet E. 662 feet

2. Let $A = \{1, 2, 3, 4\}$. Let M = the number of distinct proper subsets of A. Let N = the number of distinct differences that can be found by subtracting two distinct elements of A (for example, 1 would be one such difference since $3 - 2 = 1$). Find $M + N$.
A. 18 B. 19 C. 21 D. 22 E. 23

3. How many different ways can a cashier break (return an equivalent dollar amount in smaller denominations) a $50 bill if there are an unlimited number of $20, $10, $5, and $1 bills available to the cashier? Assume bills of the same denomination are indistinguishable.
A. 42 B. 47 C. 48 D. 56 E. 61

4. An isosceles triangle has two sides of length 40 and a base of length 48. A circle circumscribes the triangle. What is the radius of the circle?
A. $20\sqrt{2}$ B. 28 C. $18\sqrt{3}$ D. $12\sqrt{5}$ E. 25

5. Let M be the number of digits {0, 1} required to express the largest prime factor of 2019 in base 2. Let N be the number of hex digits {0, 1, 2, … E, F} required to express 2019 in base 16. Find $M - N$.
A. 7 B. 6 C. 5 D. 4 E. 3

6. Triangle ABC has vertices at (8,8), (6,4), and (10, 7). Find the sum of the lengths of the three altitudes of this triangle, rounded to the nearest tenth.
A. 8.7 B. 8.9 C. 9.2 D. 9.5 E. 9.7

7. The polynomial $2x^3 + x^2 + cx + d$ is divisible by $x + 1$. If d and c are integers with $d + c = 29$, find the sum of the two non-real roots of this polynomial.
A. -1 B. -1/2 C. 0 D. 1/2 E. 1

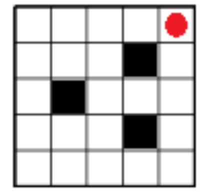
8. Let N be the smallest integer greater than 2 such that N^{N-1} is not the square of an integer. Find the product of all rational numbers that could be roots of $5x^4 + bx^3 + cx^2 + dx + N$, where b, c, and d can be any integers. Round your answer to the nearest hundredth.
A. 0 B. 2.07 C. 3.14 D. 4.30 E. 6.22

9. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says "I am a spy." Y says "X is telling the truth." Z says "I am not a spy." Which of the following correctly identifies all three people?

A.	B.	C.	D.	E.
X is the spy.	X is the spy.	X is the knight.	X is the knight.	X is the knave.
Y is the knight.	Y is the knave.	Y is the knave.	Y is the spy.	Y is the spy.
Z is the knave.	Z is the knight.	Z is the spy.	Z is the knave.	Z is the knight.

10. Morse code involves transmitting dots "•" and dashes "▬". An agent attempted to send a five-character code five different times, but only one of the five transmissions was correct. However, it was known that each erroneous transmission had a different number of errors than the others, and no transmission had five errors. The first transmission was • ▬ ▬ • ▬, which was not correct. The other four transmissions are listed below. Which one is correct?
A. • ▬ ▬ ▬ • B. ▬ ▬ • • ▬ C. • ▬ • ▬ • D. • • • • • E. Impossible to determine

11. A checker is placed on a 5×5 checkerboard as pictured. The checker may be moved one square at a time but only to the left or down. Also, the checker may not move to any of the three black squares. In how many different ways can the checker be moved to the lower left corner of the board?



A. 7 B. 8 C. 9 D. 10 E. 12

12. The function $P(t) = \cos(8t)$ can be written as sums and differences of powers of cos t only. When $P(t)$ is written this way, what is the coefficient of $(\cos t)^3$?

A. -2 B. -1 C. 0 D. 1 E. 2

13. Find the length of the shortest line segment with one endpoint on the line passing through (7, 8) that is parallel to the vector <3, -4> and the other endpoint on the circle with equation $(x + 3)^2 + (y - 2)^2 = 3$. Round your answer to the nearest tenth.

A. 9.6 B. 9.9 C. 10.0 D. 10.2 E. 11.3

14. Let a and b be positive integers with $a^2 + b^2 = 2019^2$. Find $a + b$.

A. 2319 B. 2540 C. 2711 D. 2719 E. 2811

15. Which describes the graph (in $\mathbb{R}^3$) of all solutions of this system? $\begin{cases} 2x - 6y - 8z = 15 \\ -8x - 8y + 6z = -65 \\ x - 19y - 17z = 5 \end{cases}$

A. A point B. A line C. Two lines D. A plane E. Two planes

16. The graph of $f(x) = ax^2 + bx + c$ is symmetric about the y-axis and its x and y intercepts form an equilateral triangle. If the maximum value of $f(x)$ is 4, find $a + b + c$.

A. -13/4 B. -3/4 C. 3/4 D. 13/4 E. 53/16

17. How many of the following are both a continuous function on $\mathbb{R}$ and also one-to-one?
$g(x) = \ln(e^x)$, $h(x) = x|x|$, $k(x) = x^2$, $m(x) = \frac{1}{x+1}$, $n(x) = \frac{x}{x^2+1}$, $p(x) = \sin(x)$, $q(x) = \arctan(x)$,
$r(x) = \frac{x}{|x|+1}$.

A. 2 B. 3 C. 4 D. 5 E. 6

18. Let $\{a_n\}$ be an arithmetic sequence with initial value m and common difference d. Let $\{g_n\}$ be a geometric sequence with initial value k and common ratio 2. The sum of the first 100 terms of $\{a_n\}$ and the sum of the first 10 terms of $\{g_n\}$ are equal. If m, d, and k are all positive integers, which of the following numbers must divide m?

A. 2 B. 5 C. 17 D. 31 E. 33

19. Some hikers set out on a hike at noon. At some point, they turn around and follow the same path back to where they began, and arrive there at 8:00 p.m. Their speed is 4 mi/hr on level ground, 3 mi/hr uphill and 6 mi/hr downhill. How many miles did they hike?

A. 36 B. 32 C. 28 D. 24 E. Impossible to determine

20. In the game of craps, a player (known as the shooter) rolls two fair six-sided dice. The shooter immediately loses if the sum of the dice is 2, 3, or 12 and immediately wins if the sum of the dice is 7 or 11 on the first roll. If the sum is anything else (4, 5, 6, 8, 9, or 10), that number becomes the *point* and the shooter rolls again. The shooter now wins by rolling that same point again and loses by rolling a 7. If any other number is rolled, the shooter rolls again and keeps rolling until the shooter wins by rolling the point or loses by rolling a 7. Find the probability that the shooter wins.

A. 17/36 B. 187/385 C. 244/495 D. 107/216 E. 647/1296

1. C
2. C
3. D
4. E
5. A
6. A
7. D
8. D
9. E
10. A
11. D
12. C
13. B
14. E
15. B
16. D
17. C
18. E
19. B
20. C